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SOFTWARE FOR CALCULATING AREAL SURFACE TEXTURE FUNCTION AND FEATURE PARAMETERS: USER MANUAL

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ABSTRACT

This report constitutes a *user manual* for software developed at the National Physical Laboratory for calculating S–F and S–L areal surface texture function and feature parameters. The software is used as a benchmark against which software in a measuring instrument can be compared. A data set is used as input to both this software and the software under test, and the results delivered are compared. The software is provided with a simple-to-use graphical user interface and the calculated surface texture parameter values are written to an output text file.

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Contents

1 Introduction

1.1 Background

Knowledge of the topography of a machined surface is necessary in order to understand the functional performance of the surface, and is consequently essential to the manufacturing process [1, 2, 3]. As with the instruments used in many other areas of metrology, those that measure surface texture generally interface to a computer that collects and analyses numerical data. As part of the data analysis, surface texture parameters are evaluated. The use of a surface texture parameter to characterise numerically the measured topography was proposed many years ago [4], and a number of parameters have been adopted by standards bodies and are used in industry for this purpose. Whereas it is commonplace to use physical artefacts to calibrate surface measuring instruments, such artefacts do not allow validation and verification in isolation of the software components of the instrument.

ISO 5436–2 [5] introduced into international standardisation the concept of the *software measurement standard* in the context of the measurement of surface texture. The standard defines Type F1 (reference data) and Type F2 (reference software) software measurement standards for testing the numerical correctness of software used in surface texture measurement. Reference software is used as a benchmark against which software in a measuring instrument can be compared. A data set is used as input to both the software under test and the reference software, and the results delivered by the software under test are compared with those provided by the reference software. The most important consideration in the design and development of reference software is its numerical correctness. This is in contrast to the considerations for production software, for which the requirements on numerical correctness are generally more modest, but issues of efficiency, such as computing time and memory, and usability are of concern.

Previous work [6, 7, 8] described software for the calculation of S–F and S–L areal surface texture parameters with a focus on *height* and *hybrid* parameters. This report is a user manual for software developed at the National Physical Laboratory (NPL) with support from the University of Huddersfield for the calculation of S–F and S–L areal surface texture *function* and *feature* parameters. The software is supplemented by a paper [9] that describes the software, focusing on the definitions of the surface texture parameters and describing the numerical algorithms employed in the software to implement those definitions. Additionally, the paper presents a comparison of the results returned by the software with those obtained using proprietary software for surface texture measurement.

The report is organised as follows. The remainder of this section describes the main functions of the software, including the conditions for its use. Section 2 gives information on installing and uninstalling the software. Section 3 describes how to use the software. Finally, appendices to the report contain technical information on the definitions of the areal surface texture function and feature parameters considered.

1.2 Software User Licence Agreement

The software is provided with a Software User Licence Agreement (Ref: MSC/L/15/001) and the use of the software is subject to the terms laid out in that agreement. By installing and running the software, the user accepts the terms of the agreement.

To run the software, the user must first install MATLAB's Component Runtime (MCR) libraries (section 2). The MCR installer file is not included in the software distribution, but it may be obtained by contacting NPL. Information on how to obtain the MCR installer file is given in the file `README.txt` included in the software distribution. The user must accept the terms of the MCR Library License as part of the installation of the MCR libraries.

1.3 Main functions of the software

The main functions of the software are as follows:

1. Read data defining an *S–F surface* [10, clause 3.1.5], *i.e.* a surface derived by applying an F–operator to remove form and an S–filter to remove small scale components;

2. Apply a Gaussian areal filter to the data defining an S–F surface to obtain data defining the corresponding *S–L surface* [10, clause 3.1.6] (appendix A);

3. Remove points from the data defining an S–L surface (*i.e.* 'trim' the data) to produce a surface with a corresponding evaluation area which has sides with lengths that are integer multiples of the cut-off wavelength for the Gaussian areal filter used to obtain the surface (appendix B);

4. For *function* parameters (appendix C.2):

   (a) Evaluate approximations to the inverse areal material ratio functions for the S–F and S–L surfaces in the form of (shape-preserving) cubic spline interpolants to data defining those functions;

   (b) Evaluate function parameters in terms of those representations of the inverse areal material ratio functions;

5. For *feature* parameters (appendix C.3):

   (a) Apply segmentation to data defining the S–F and S–L surfaces to identify significant features of those surfaces;

   (b) Evaluate feature parameters in terms of those significant features of the S–F and S–L surfaces;

6. Write the values of the areal surface texture function and feature parameters to an output file.

The areal surface texture parameters to be evaluated include *function* parameters

- core height S_{k},
- peaks material ratio S_{mr1},
- dales material ratio S_{mr2},
- reduced peak height S_{pk},
- reduced dale height S_{vk},
- peak extreme height S_{xp},
- peak material volume V_{mp},
- core material volume V_{mc},
- dale void volume V_{vv},
- core void volume V_{vc},

and *feature* parameters

- density of peaks S_{pd},
- arithmetic mean peak curvature S_{pc},
- five-point peak height S_{5p},
- five-point pit height S_{5v},
- ten-point height of surface S_{10z},
- mean dale area $S_{\mathrm{da}}(c)$,
- mean hill area $S_{\mathrm{ha}}(c)$,
- mean dale volume $S_{\mathrm{dv}}(c)$,
- mean hill volume $S_{\mathrm{hv}}(c)$.

The following conditions are assumed to apply:

1. The input data is provided in X3P format [11], a format for defining surface data that allows the smooth interchange of data between different measuring systems. Software implementing the data format has been developed by the 'openGPS' consortium as freeware [12];

2. The input data has a uniform sampling inverval $\Delta x > 0$ in the x–direction and a uniform sampling interval $\Delta y > 0$ in the y–direction with, in general, $\Delta x \neq \Delta y$;

3. For an S–F surface, the *evaluation area* [10, clause 3.1.10], *i.e.* the area used to specify the part of the surface under evaluation, is a rectangle with sides of lengths L_x in the x–direction and L_y in the y–direction containing all the surface heights;

4. For Gaussian areal filtering, the same cut-off wavelength $\lambda > 0$ is applied in the x– and y–directions;

5. The cut-off wavelength λ is an integer multiple of the sampling intervals Δx and Δy;

6. In order to minimise any distortion of the filtered surface due to the finite extent of the surface, the lengths L_x and L_x are at least three times the cut-off wavelength;

7. In order to minimise any distortion of the filtered surface due to the finite number of data points per cut-off wavelength, there should be at least fifty points per cut-off wavelength in each direction;

8. For an S–L surface, the evaluation area is a rectangle of largest area with sides having lengths equal to integer multiples of λ that is contained within the rectangle obtained from the evaluation area for the corresponding S–F surface by removing λ from each side. The evaluation area for an S–L surface is generally not uniquely defined. A particular choice is made by requiring that it is positioned 'centrally' with respect to the evaluation area for the corresponding S–F surface (appendix B).

For evaluating surface texture parameters for an S–F surface, the input data consists of matrices $\boldsymbol{X}$, $\boldsymbol{Y}$ and $\boldsymbol{Z}$, each of dimension $m_x \times m_y$, containing, respectively, the x–coordinates $x_{ij} = x_0 + (i-1)\Delta_x$, y–coordinates $y_{ij} = y_0 + (j-1)\Delta_y$ and surface heights (z–coordinates) z_{ij}, $i = 1, \ldots, m_x$, $j = 1, \ldots, m_y$, *i.e.* z_{11} is the height at the origin (x_0, y_0) of the xy–coordinate system and $z_{m_x m_y}$ is the height at the point 'diagonally opposed' to the origin. For evaluating surface texture parameters for an S–L surface, the input data consists of that for an S–F surface together with the cut-off wavelength λ.

It is assumed that the x–, y– and z–coordinates, together with the cut-off wavelength λ, are provided in the same unit of measurement, which then determines the units of measurement of the calculated surface texture parameters. For example, if the data is provided with $\mu\mathrm{m}$ as the unit of measurement and the scaling factor K (section 3.3.5) is assigned a value of 1, then the calculated surface texture parameters have the following units of measurement:

S_{k}	$\mu\mathrm{m}$
S_{mr1}	%
S_{mr2}	%
S_{pk}	$\mu\mathrm{m}$
S_{vk}	$\mu\mathrm{m}$
S_{xp}	$\mu\mathrm{m}$
V_{mp}	$\mathrm{ml/m^2}$
V_{mc}	$\mathrm{ml/m^2}$
V_{vv}	$\mathrm{ml/m^2}$
V_{vc}	$\mathrm{ml/m^2}$
S_{pd}	$\mu\mathrm{m}^{-2}$
S_{pc}	$\mu\mathrm{m}^{-2}$
S_{5p}	$\mu\mathrm{m}$
S_{5v}	$\mu\mathrm{m}$
S_{10z}	$\mu\mathrm{m}$
$S_{\mathrm{da}}(c)$	$\mu\mathrm{m}^2$
$S_{\mathrm{ha}}(c)$	$\mu\mathrm{m}^2$
$S_{\mathrm{dv}}(c)$	$\mu\mathrm{m}^3$
$S_{\mathrm{hv}}(c)$	$\mu\mathrm{m}^3$

2 Installing and uninstalling the software

The software takes the form of an application program called

```
FunctionFeatureCharacterisationSoftware.exe
```

generated by compiling (using the MATLAB compiler) software implemented in the MATLAB programming language [13], together with the associated files:

- `ISO5436_2_XML.dll`;
- `xerces-c_2_7_vc80.dll`;
- `tbb.dll`;
- `openX3P.mexw32`;
- `writeX3P.mexw32`;

- `iso5436_2.xsd`;
- `npl_logo_black_large.jpg`;
- `FunctionFeatureCharacterisationSoftware-R1.pdf`;
- `FunctionFeatureCharacterisationSoftware-MSC_L_15_001.pdf`;
- `HudHoned512_LS_scaled.x3p`;
- `HudHoned512_LS_scaled_FF.txt`;
- `README.txt`.

The program has been created and tested on a personal computer running the Microsoft Windows 7 operating system.

To install the software, undertake the following steps:

1. Copy the application program and the above associated files to a working folder;
2. Create the User Environment Variable `OPENGPS_LOCATION` and set its value to the path of the working folder containing the application program and its associated files;
3. Install MATLAB's Component Runtime (MCR) libraries. This is done by running the MCR installation program

   ```
   MCRInstaller.exe
   ```

   once on the target machine, *i.e.* the machine on which it is intended to run the application program. It is necessary to have administrative privileges for the target machine because both the system registry and system path are modified as part of the installation process. The MCR installation program installs the MCR libraries, registers the components as needed, and updates the system path to point to the MCR binary folder. The installation process takes some time due to the number of files that are installed. The MCR installation program is about 537 MB in size, and the installed libraries require about 1.24 GB of disk space.

The software is uninstalled by

- running the MCR installation program `MCRInstaller.exe` and selecting 'Remove' to uninstall the MCR libraries, and
- deleting the application program and its associated files.

3 Using the software

3.1 General

The application program may be run in either of two ways:

(a) Double-clicking on the executable file (with the extension `.exe`) in Windows Explorer. An MS-DOS window opens and the main graphical user interface (GUI) is displayed;

(b) Opening an MS-DOS window, navigating to the folder containing the program, typing the name of the program (without the extension `.exe`), and pressing `Return`. The main GUI is displayed (and the MS-DOS command line may not be accessed again until the user has exited the software).

It may take several seconds for the GUI to be displayed.

The GUI is divided into a number of parts that allow the user to perform the following operations:

1. Select X3P file;
2. Select (surface texture) parameters;
3. Select output file;
4. Process data;
5. Exit.

Each of these operations is described in the sections below.

3.2 Select X3P file

The user must first select an X3P file. This is done by pressing the `BROWSE...` button in part 1 of the GUI, navigating to the folder in which the required X3P file is located and either double-clicking on that file or clicking on the file and pressing `Open`. If no file has been selected and the user presses `Open`, nothing happens. If the user presses `Cancel` (with or without selecting an X3P file), a warning message is displayed.

3.3 Select surface texture parameters

Once the user has selected an X3P file (section 3.2), the following components in part 2 of the GUI become active:

- check box ‘S-F FUNCTION PARAMETERS’,
- check box ‘S-F FEATURE PARAMETERS’,
- check box ‘S-L FUNCTION PARAMETERS’,
- check box ‘S-L FEATURE PARAMETERS’,
- edit box ‘Number of height values for Smr curve’ (section 3.3.1), and
- edit box ‘Number of interpolation points for Smr curve’ (section 3.3.2).

To calculate function parameters for the S–F surface, the user clicks on the check-box ‘S-F FUNCTION PARAMETERS’ (repeated clicking on any check-box toggles between the check-box being checked or not checked). The following components in part 2 of the GUI become active:

- edit box ‘Number of interpolation points for calculation of equivalent straight line’ (section 3.3.4), and
- edit box ‘Scaling factor K’ (section 3.3.5).

The user may choose to use or edit the default values.

To calculate feature parameters for the S–F surface, the user clicks on the check-box ‘S-F FEATURE PARAMETERS’. The following components in part 2 of the GUI become active:

- menu ‘Dale designation’ (section 3.3.6).
- edit box ‘Dale threshold’ (section 3.3.7).
- menu ‘Hill designation’ (section 3.3.8).
- edit box ‘Hill threshold’ (section 3.3.9).
- edit box ‘Number of interpolation points for calculation of mean areas’ (section 3.3.10), and
- edit box ‘Number of interpolation points for calculation of volume areas’ (section 3.3.11).

The user may choose to use or edit the default values.

The calculations of function and feature parameters for the S–L surface are selected similarly to the S–F surface. If the user clicks on either of the check boxes ‘S-L FUNCTION PARAMETERS’ or ‘S-L FEATURE PARAMETER’, the following component in part 2 of the GUI becomes active:

- edit box ‘Cut-off wavelength’ (section 3.3.3).

3.3.1 Set number of height values for Smr curve

For the calculation of all surface texture parameters, the user is required to provide a value n_{mr} of the number of height values in a discrete representation of the S_{mr} curve (appendix C.2.1). Increasing the value for n_{mr} increases the computation time.

The default value for n_{mr} is 101.

A check is made that the value n_{mr} entered by the user is an integer greater than or equal to 2.

3.3.2 Set number of interpolation points for Smr curve

For the calculation of all surface texture parameters, the user is required to provide a value n_{i} of the number of interpolation points used to obtain an approximation to each value $S_{\mathrm{mr}}(c_k),\ k = 1, \ldots, n_{\mathrm{mr}}$, in a discrete representation of the S_{mr} curve (appendix C.2.1). A large value for n_{i} improves the quality of the approximation but increases the computation time. An indication of the quality of the approximation is given in the output file (section 3.8).

The default value for n_{i} is 101.

A check is made that the value n_{i} entered by the user is an integer greater than or equal to to 2.

3.3.3 Set cut-off wavelength

For the calculation of surface texture parameters for an S–L surface, the user is required to provide a value λ of the cut-off wavelength of an areal Gaussian filter.

The default value for λ is 100, which corresponds to 0.1 mm when the x–, y– and z–coordinates are provided in units of micrometres.

Checks are made that:

- λ is greater than zero;
- $\lambda/\Delta x$ is an integer greater than or equal to 50, where Δx is the uniform sampling interval in the x-direction;
- $\lambda/\Delta y$ is an integer greater than or equal to 50, where Δy is the uniform sampling interval in the y-direction;
- L_x/λ is greater than or equal to 3, where L_x is the length of the definition area in the x-direction;

- L_y/λ is greater than or equal to 3, where L_y is the length of the definition area in the y-direction.

3.3.4 Set number of interpolation points for calculation of equivalent straight line

For the calculation of function parameters, the user is required to provide a value n_{p} of the number of interpolation points used to obtain an approximation to the equivalent straight line (appendix C.2.3). A large value for n_{p} improves the quality of the approximation but increases the computation time.

The default value for n_{p} is $1\,001$.

A check is made that the value n_{p} entered by the user is an integer greater than or equal to to 2.

3.3.5 Set scaling factor

For the calculation of function parameters, the user is required to provide a value K of the scaling factor used to convert the units of the (volume) parameters V_{mp}, V_{mc}, V_{vv} and V_{vc} to millilitres per square metre.

The default value for K is 1, corresponding to the case when the data is provided with $\mu\mathrm{m}$ as the unit of measurement.

A check is made that the value K entered by the user is positive.

3.3.6 Set dale designation

For the calculation of feature parameters, the user is required to select a dale designation.

The default value of dale designation is ‘Closed’.

3.3.7 Set dale threshold

For the calculation of feature parameters, the user is required to select a value D_{c} of the dale threshold.

The default value for D_{c} is $0\,\%$.

A check is made that the value D_{c} entered by the user lies in the closed interval $[0\,\%, 100\,\%]$.

3.3.8 Set hill designation

For the calculation of feature parameters, the user is required to select a hill designation.

The default value of hill designation is 'Closed'.

3.3.9 Set hill threshold

For the calculation of feature parameters, the user is required to select a value H_c of the hill threshold.

The default value for H_c is $100\,\%$.

A check is made that the value H_c entered by the user lies in the closed interval $[0\,\%, 100\,\%]$.

3.3.10 Set number of interpolation points for calculation of mean areas

For the calculation of feature parameters, the user is required to select a value n_ia of the number of interpolation points used to obtain an approximation to a (mean) area. A large value for n_ia improves the quality of the approximation but increases the computation time. An indication of the quality of the approximation is given in the output file (section 3.8).

The default value for n_ia is 11.

A check is made that the value n_ia entered by the user is an integer greater than or equal to to 2.

3.3.11 Set number of interpolation points for calculation of mean volumes

For the calculation of feature parameters, the user is required to select a value n_iv of the number of interpolation points used to obtain an approximation to a (mean) volume. A large value for n_iv improves the quality of the approximation but increases the computation time. An indication of the quality of the approximation is given in the output file (section 3.8).

The default value for n_iv is 11.

A check is made that the value n_iv entered by the user is an integer greater than or equal to to 2.

3.4 Select output file

Once the user has selected an X3P file (section 3.2), a default output file path appears in the edit box in part 3 of the GUI. The default output file is located in the same folder as the X3P file. Its base name is that of the X3P file with `_FF` appended and it has the extension `.txt`. The user may edit the default output file path, including the extension of the file name, which is not required to be `.txt`. If the file does not exist, it will be created. If it does exist, results will be appended to the end of the file.

3.5 Process data

When at least one check box in part 2 of the GUI has been ticked, the `CALCULATE` button in part 4 of the GUI becomes active.

To calculate the selected surface texture parameters, press the `CALCULATE` button. A 'progress' text box appears that indicates the current calculation being undertaken. When this text box disappears, all calculations have been completed.

If an edit box in part 2 of the GUI is empty when it should contain a value, a warning message is displayed and the software does not attempt to undertake any calculation.

3.6 Exit

Pressing the `EXIT` button in part 5 of the GUI terminates the program and closes the GUI and any figures generated during processing (section 3.9). Alternatively, the user may click on 'File' in the main menu of the GUI and then click on 'Exit'.

If the application program was run by double-clicking on the executable file (section 3.1, (a)), the MS-DOS window closes. If it was run from within an MS-DOS window (section 3.1, (b)), the user may now access the MS-DOS command line again.

It may take several seconds for the 'Application Error' message to be displayed.

3.7 Help

Clicking on 'Help' in the main menu of the GUI and then clicking on 'Software Details' causes a message box displaying information about the software to be displayed. Click on 'OK' to close this message box.

Clicking on 'Help' in the main menu of the GUI and then clicking on 'Help Documentation' causes the user manual (this document) to be opened (using the default program for viewing PDF files).

Clicking on 'Help' in the main menu of the GUI and then clicking on 'Licence Agreement' causes the licence agreement to be opened (using the default program for viewing PDF files).

3.8 Output file

Each time the `CALCULATE` button is pressed, the following information is written to the output file:

- Details (name, release number and date of release) of the software;
- The date and time at which the software was run;
- The path to the X3P file being processed;
- If S–F surface is selected:
  - An indication ('maximum deviation') of the quality of the approximation to each value $S_{\rm mr}(c_k)$, $k = 1, \ldots, n_{\rm mr}$, in a discrete representation of the $S_{\rm mr}$ curve;
  - The values of function parameters (if selected);
  - The values of feature parameter (if selected);
- If S–L surface is selected:
  - An indication ('maximum deviation') of the quality of the approximation to each value $S_{\rm mr}(c_k)$, $k = 1, \ldots, n_{\rm mr}$, in a discrete representation of the $S_{\rm mr}$ curve;
  - The values of function parameters (if selected);
  - The values of feature parameter (if selected).

When a value for the parameter $S_{\rm 5p}$, $S_{\rm 5v}$ or $S_{\rm 10z}$ is not returned (because there are insufficient numbers of hill or dale regions), a 'NaN' is displayed in the output file.

For the parameters $S_{\rm da}$, $S_{\rm ha}$, $S_{\rm dv}$ and $S_{\rm hv}$, a value for the parameter is displayed together with an indication of its quality.

Results are presented using 16 significant decimal digits of precision. Users can then compare the results with those returned by other software using a number of digits that is considered to be meaningful.

The output file records the values of all parameters set by the user that are used to control the calculations undertaken.

3.9 Output figures

The following output figures are produced by the software:

- For an S–F (or S–L) surface, graphs showing the data defining a discrete representation of the S_{mr} curve for the surface (appendix C.2.1) and the approximation to the S_{mc} function based on that data (appendix C.2.2);
- If feature parameters are selected, a visualisation of the result of applying segmentation to the S–F (or S–L) surface, showing the regions corresponding to the significant hills and dales of the surface (appendix C.3.2).

3.10 Example files

The example X3P file

```
HudHoned512_LS_scaled.x3p
```

is provided by the University of Huddersfield as part of the software distribution. Also included is the example output file

```
HudHoned512_LS_scaled_FF.txt
```

which was generated by processing the example X3P file, calculating all S–F and S–L function and feature parameters using the default values for all components for which values must be provided by the user.

Having installed the software, users may run it on the example X3P file to check that installation has been successful. The values of the parameters should agree with those in the example output file.

Acknowledgements

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A Gaussian areal filter

The Gaussian areal filter is defined by the weighting function

$$s_{x,y}(\xi,\eta) = \frac{1}{\alpha^2\lambda^2}\exp\left[-\pi\left(\frac{\xi}{\alpha\lambda}\right)^2 - \pi\left(\frac{\eta}{\alpha\lambda}\right)^2\right],$$

with $\alpha = \sqrt{\log 2/\pi}$, and λ denoting the cut-off wavelength in the x– and y–directions [14]. Given a continuous surface $z(x,y)$, the Gaussian filtered surface $w(x,y)$ is defined by the convolution integral

$$w(x,y) = \int\int z(\xi,\eta)s_{x,y}(x-\xi, y-\eta)\,\mathrm{d}\xi\mathrm{d}\eta.$$

Due to the separability of the weighting function

$$w(x,y) = \int\left[\int z(\xi,\eta)s(x-\xi)\,\mathrm{d}\xi\right]s(y-\eta)\,\mathrm{d}\eta,$$

where

$$s(\xi) = \frac{1}{\alpha\lambda}\exp\left[-\pi\left(\frac{\xi}{\alpha\lambda}\right)^2\right]$$

is the weighting function for a Gaussian profile filter with cut-off wavelength λ [15, 16]. It follows that Gaussian filtration for an areal surface can be undertaken by applying profile Gaussian filtration in the x–direction followed by profile Gaussian filtration in the y–direction (or *vice versa*).

In practice, knowledge of the surface $z(x,y)$ takes the form of surface heights z_{ij} at finitely-many points (x_i, y_j). Then, the calculation of w_{ij}, an approximation to $w(x_i, y_j)$, is implemented using the discrete convolution

$$w_{ij} = \sum_{\ell=j-n_y}^{j+n_y}\left[\sum_{k=i-n_x}^{i+n_x} z_{k\ell}s(x_i-x_k)\right]s(y_j-y_\ell)$$

for $i = 1 + n_x, \ldots, m_x - n_x,\ j = 1 + n_y, \ldots, m_y - n_y$, and integers $n_x = \lambda/\Delta_x$ and $n_y = \lambda/\Delta_y$. Finally, the S–L surface derived from the S–F surface by Gaussian areal filtration is defined by surface heights r_{ij} with

$$r_{ij} = z_{ij} - w_{ij}, \quad i = 1 + n_x, \ldots, m_x - n_x, \quad j = 1 + n_y, \ldots, m_y - n_y.$$

B Evaluation area for an S–L surface

Following application of a Gaussian areal filter, the S–L surface is defined by surface heights $r_{ij},\ i = 1+n_x, \ldots, m_x - n_x,\ j = 1+n_y, \ldots, m_y - n_y$, corresponding to points (x_i, y_j) in a rectangle with sides of lengths $L_x - 2\lambda$ in the x–direction and $L_y - 2\lambda$ in the y–direction. In general, the lengths are not integer multiples of λ, and it is necessary to trim the data set (x_i, y_j, r_{ij}).

Consider first the x–direction. Let $H_x = p_x\lambda$, where p_x is chosen to be the largest integer for which $H_x \leq L_x - 2\lambda$. Since $\lambda = n_x\Delta x$ for integer n_x (appendix A), $H_x = p_x n_x \Delta x$. Consequently, the length $L_x - 2\lambda$ contains $m_x - 2n_x$ points and the length H_x contains $p_x n_x + 1$ points. Define

$$r_x = (m_x - 2n_x) - (p_x n_x + 1) = m_x - (2 + p_x)n_x - 1.$$

If r_x is even, then $r_x/2$ points x_i are removed from the left- and right-hand ends of the sets $x_i,\ i = 1 + n_x, \ldots, m_x - n_x$. If r_x is odd, then int$(r_x/2)$ points are removed from the left-hand end, and int$(r_x/2) + 1$ points from the right-hand end, where int(q) denotes the integer part of q.

Similar considerations apply to the y–direction. After removing points in this way, the resulting set of heights r_{ij} defines the S–L surface for which areal surface texture parameters are calculated, and the corresponding set of points (x_i, y_j) defines the evaluation area for that S–L surface, which is a rectangle with sides of lengths H_x in the x–direction and H_y in the y–direction.

C Definitions of areal surface texture parameters

C.1 Surface representation

In the definitions of the function parameters (appendix C.2) and the feature parameters (appendix C.3), $f(x, y) \equiv z(x, y)$ and $f_{ij} \equiv z_{ij}$ for an S–F surface or $f(x, y) \equiv r(x, y)$ and $f_{ij} \equiv r_{ij}$ for an S–L surface. For an S–F (or S–L) surface, the evaluation area A is expressed as the union of rectangular regions $A_{ij},\ i = 1, \ldots, m_x - 1,\ j = 1, \ldots, m_y - 1$, defined by the locations (x_i, y_j) of the measured heights for the surface:

$$A = \bigcup_{ij} A_{ij},$$

where

$$A_{ij} = \{(x, y) : x_i \leq x \leq x_{i+1}, y_j \leq y \leq y_{j+1}\},$$

and A_{ij} has area

$$|A_{ij}| = \Delta x \Delta y.$$

On each rectangular region A_{ij}, the function f is here defined to be the bilinear surface that interpolates the surface heights $f_{ij},\ f_{i+1,j},\ f_{i,j+1}$ and $f_{i+1,j+1}$ at the vertices of the region. In terms of normalized variables

$$X = \frac{x - x_i}{x_{i+1} - x_i} = \frac{x - x_i}{\Delta x}, \quad Y = \frac{y - y_j}{y_{j+1} - y_j} = \frac{y - y_j}{\Delta y},$$

the function f can be written as

$$f(X, Y) = (1 - X)(1 - Y)f_{ij} + X(1 - Y)f_{i+1,j} + (1 - X)Y f_{i,j+1} + XY f_{i+1,j+1}. \quad (1)$$

It follows that the volume $|V_{ij}|$ defined by the function f and the rectangular region A_{ij} is

$$|V_{ij}| = \frac{\Delta x \Delta y}{4} \left(f_{i,j} + f_{i+1,j} + f_{i,j+1} + f_{i+1,j+1}\right).$$

The definitions for a number of the areal surface texture parameters considered below are expressed in terms of the area $|A(c)|$ of the region $A(c)$ corresponding to that part of an S–F or S–L surface with height greater than or equal to a specified value c, *i.e.*

$$|A(c)| = \iint_{(x,y)\in A(c)} \mathrm{d}x\mathrm{d}y = \sum_{i,j} \iint_{(x,y)\in A_{ij}(c)} \mathrm{d}x\mathrm{d}y,$$

where

$$A(c) = \{(x, y) \in A : f(x, y) \geq c\}, \quad A_{ij}(c) = \{(x, y) \in A_{ij} : f(x, y) \geq c\}.$$

Similarly, a number of those definitions are expressed in terms of the volume $|V(c)|$ defined by an S–F or S–L surface $f(x, y)$ and the region $A(c)$, *i.e.*

$$|V(c)| = \iint_{(x,y)\in A(c)} f(x, y)\, \mathrm{d}x\mathrm{d}y = \sum_{i,j} \iint_{(x,y)\in A_{ij}(c)} f(x, y)\, \mathrm{d}x\mathrm{d}y.$$

C.2 Function parameter evaluation

C.2.1 Areal material ratio (function)

The areal material ratio $S_{\rm mr}(c)$ of a scale limited surface is defined as the ratio of the area of the material at a specified height c to the evaluation area [10, clause 4.4.1, 4.4.2]:

$$S_{\rm mr}(c) = \frac{|A(c)|}{|A|}.$$

A discrete representation of the areal material ratio function is obtained by calculating the areal material ratio $S_{{\rm mr},k} \equiv S_{\rm mr}(c_k)$ for a set of discrete, uniformly spaced, heights c_k, $k = 1, \ldots, n_{\rm mr}$.

C.2.2 Inverse areal material ratio (function)

The areal material ratio function $S_{\rm mr}(c)$ is a monotonic function and therefore its inverse, referred to as the inverse areal material ratio function, exists. The inverse areal material ratio $S_{\rm mc}(p)$ of a scale limited surface is defined as the height at which the ratio of the area of the material at that height to the evaluation area is p [10, clause 4.4.3]. An approximation $c = S_{\rm mc}(p)$ to the inverse areal material ratio function is here constructed as a shape-preserving piecewise cubic polynomial interpolant to the data points $(p_k = S_{{\rm mr},k}, c_k),\ k = 1, \ldots, n_{\rm mr}$, obtained from the evaluation of the areal material function.

C.2.3 Equivalent straight line for inverse areal material ratio function

The equivalent straight line is required for the calculation of core height, peaks material ratio, dales material ratio, reduced peak height and reduced dale height. It corresponds to the areal material ratio $p_{\rm E}$ such that the (absolute) gradient of the secant joining the points $(p_{\rm E}, S_{\rm mc}(p_{\rm E}))$ and $(p_{\rm E} + 40\ \%, S_{\rm mc}(p_{\rm E} + 40\ \%))$ is the smallest of all the (absolute) gradients of the secants joining the points $(p, S_{\rm mc}(p))$ and $(p + 40\ \%, S_{\rm mc}(p + 40\ \%))$, $0\ \% \leq p \leq 60\ \%$ [10, clause 5.2]. The equivalent straight line is then defined to be that secant extrapolated to the interval $[0\ \%, 100\ \%]$, and can be described by the equation

$$S_{\rm esl}(p) = S_{\rm mc}(p_E) + \frac{(p - p_E)(S_{\rm mc}(p_E + 40) - S_{\rm mc}(p_E))}{40}. \tag{2}$$

The value $S_{\rm mc,H} \equiv S_{\rm esl}(0)$ of the equivalent straight line (2) that corresponds to $p = 0\ \%$ identifies the highest level of the core surface, and the value $S_{\rm mc,L} \equiv S_{\rm esl}(100)$ of the equivalent straight line (2) that corresponds to $p = 100\ \%$ identifies the lowest level of the core surface.

C.2.4 Core height

The core height S_{k} is defined as the distance between the highest and lowest levels of the core surface [10, clause 4.4.4.2]:

$$S_{\mathrm{k}} = S_{\mathrm{mc,H}} - S_{\mathrm{mc,L}}.$$

C.2.5 Peaks material ratio

The peaks material ratio S_{mr1} is defined as the ratio of the area of the material at the intersection line which separates the protruding hills from the core surface to the evaluation area [10, clause 4.4.4.5]. It corresponds to the solution of the equation

$$S_{\mathrm{mc}}(S_{\mathrm{mr1}}) = S_{mc,H},$$

which gives the value of the areal material ratio corresponding to the height $S_{\mathrm{mc,H}}$.

C.2.6 Dales material ratio

The dales material ratio S_{mr2} is defined as ratio of the area of the material at the intersection line which separates the protruding dales from the core surface to the evaluation area [10, clause 4.4.4.6]. It corresponds to the solution of the equation

$$S_{\mathrm{mc}}(S_{\mathrm{mr2}}) = S_{mc,L},$$

which gives the value of the areal material ratio corresponding to the height $S_{\mathrm{mc,L}}$.

C.2.7 Reduced peak height

The reduced peak height S_{pk} is defined as the average height of the protruding peaks above the core surface [10, clause 4.4.4.3]. It is the height of the right-angled triangle constructed to have the same area as the “hill area”, *i.e.*

$$S_{\mathrm{pk}} = \frac{\int_0^{S_{\mathrm{mr1}}} S_{\mathrm{mc}}(p) - S_{\mathrm{mc,H}}\,\mathrm{d}p}{S_{\mathrm{mr1}}/2}.$$

C.2.8 Reduced dale height

The reduced dale height S_{vk} is defined as the average height of the protruding dales below the core surface [10, clause 4.4.4.4]. It is the height of the right-angled triangle constructed to have the same area as the “dale area”, *i.e.*

$$S_{\mathrm{vk}} = \frac{\int_{S_{\mathrm{mr2}}}^{100} S_{\mathrm{mc,L}} - S_{\mathrm{mc}}(p)\,\mathrm{d}p}{(100 - S_{\mathrm{mr2}})/2}.$$

C.2.9 Peak extreme height

The peak extreme height S_{xp} is defined as the difference in heights at areal material ratios $2.5\ \%$ and $50\ \%$ [10, clause 4.4.7] and [17]:

$$S_{\mathrm{xp}} = S_{\mathrm{mc}}(2.5\ \%) - S_{\mathrm{mc}}(50\ \%).$$

C.2.10 Material volume

The material volume $V_{\mathrm{m}}(p)$ at a given material ratio p is defined as the volume of the material per unit area calculated from the (inverse) areal material ratio function [10, clause 4.4.6]:

$$V_{\mathrm{m}}(p) = \frac{K}{100} \int_0^p S_{\mathrm{mc}}(q) - S_{\mathrm{mc}}(p)\,\mathrm{d}q,$$

where K is a constant to convert to units of millilitres per metres squared (for measured data having units of $\mu\mathrm{m}$, $K = 1$).

C.2.11 Peak material volume

The peak material volume V_{mp} is defined as the material volume at material ratio $p = 10\ \%$ [10, clause 4.4.6.1] and [17]:

$$V_{\mathrm{mp}} = V_{\mathrm{m}}(10\ \%) = \frac{K}{100} \int_0^{10} S_{\mathrm{mc}}(q) - S_{\mathrm{mc}}(10)\,\mathrm{d}q.$$

C.2.12 Core material volume

The core material ratio V_{mc} is defined as the difference in material volumes at material ratios $p = 10\ \%$ and $q = 80\ \%$ [10, clause 4.4.6.2] and [17]:

$$V_{\mathrm{mc}} = V_{\mathrm{m}}(80\ \%) - V_{\mathrm{m}}(10\ \%).$$

C.2.13 Void volume

The void volume $V_{\mathrm{v}}(p)$ at a given material ratio p is defined as the volume of the voids per unit area calculated from the (inverse) areal material ratio function [10, clause 4.4.5]:

$$V_{\mathrm{v}}(p) = \frac{K}{100} \int_p^{100} S_{\mathrm{mc}}(p) - S_{\mathrm{mc}}(q)\,\mathrm{d}q,$$

where K is a constant to convert to units of millilitres per metres squared (for measured data having units of $\mu\mathrm{m}$, $K = 1$).

C.2.14 Dale void volume

The dale void volume V_{vv} is defined as the dale volume at material ratio $p = 80\ \%$ [10, clause 4.4.5.1] and [17]:

$$V_{\mathrm{vv}} = V_{\mathrm{v}}(80\ \%) = \frac{K}{100} \int_{80}^{100} S_{\mathrm{mc}}(80) - S_{\mathrm{mc}}(q)\ \mathrm{d}q.$$

C.2.15 Core void volume

The core void volume V_{vc} is defined as the difference in void volumes at material ratios $p = 10\ \%$ and $q = 80\ \%$ [10, clause 4.4.5.2] and [17]:

$$V_{\mathrm{vc}} = V_{\mathrm{v}}(10\ \%) - V_{\mathrm{v}}(80\ \%).$$

C.3 Feature parameter evaluation

C.3.1 General

The process of feature characterization is in five stages [10]:

1. Selection of the type of feature;
2. Segmentation;
3. Determining significant features;
4. Selection of feature attributes;
5. Quantification of feature attribute statistics.

The three main types of feature are areal features (hills and dales), line features (course and ridge lines) and point features (peaks, pits and saddle points). The features have the designations, respectively, of H, D, C, R, P, V and S [10, clause 6.2]. The stage of segmentation is described in section C.3.2. The stages of determining significant features, selecting feature attributes and quantification of feature attribute statistics are described for the different feature parameters in the following sections.

C.3.2 Segmentation

The following approach to segmentation is applied [18]:

1. Calculate the watershed regions, *i.e.* Maxwell dales (or hills);
2. Find the lowest (or highest) saddle point on the boundary of each dale (or hill);
3. Within each watershed region calculate the height difference between the lowest pit (or highest peak) and the lowest (or highest) saddle on the boundary: this is the local pit (or local peak);
4. If this local pit (or local peak) height is below a given threshold (the height discrimination) then the watershed region is combined with that region to which the lowest (or highest) saddle is connected;
5. When all remaining local pit (or local peak) heights of the combined watershed regions are above the specified height discrimination, the combined watershed regions are the required segmentation of the height matrix at the given Wolf pruning height discrimination.

Software for applying segmentation to a matrix of surface values was provided by the University of Huddersfield, and forms part of the software described in this report.

C.3.3 Density of peaks

The density of peaks S_{pd} has the following designation [10, clause 6.8.1]:

S_{pd} = FC;H;Wolfprune:X%;All;Count;Density

that is interpreted as follows:

1. Select hills (H);
2. Apply (hill) segmentation by Wolf pruning using a threshold of X % of S_{z} [6] and [10, clause 4.1.6];
3. Select all hills;
4. Set the attribute value for each selected hill to be one;
5. Evaluate the sum of the attribute values divided by the definition area.

At step 2, the default threshold value for Wolf pruning is $5\ \%$. At step 3, only those hill regions that contain a peak strictly in the interior of the region are counted. It follows that

$$S_{\mathrm{pd}} = \frac{n_{\mathrm{h}}}{|A|},$$

where n_{h} is the number of (significant) hills and $|A|$ is the definition area (equal to the evaluation area).

C.3.4 Arithmetic mean peak curvature

The arithmetic mean peak curvature S_{pc} has the following designation [10, clause 6.8.2]:

S_{pc} = FC;P;Wolfprune:X%;All;Curvature;Mean

that is interpreted as follows:

1. Select peaks (P);
2. Apply (hill) segmentation by Wolf pruning using a threshold of X % of S_{z} [6] and [10, clause 4.1.6];
3. Select all peaks;
4. Set the attribute value for each selected peak to be the local curvature at the peak;
5. Evaluate the arithmetic mean of the attribute values.

At step 2, the default threshold value for Wolf pruning is $5\ \%$. At step 3, a peak is only counted if it is strictly within the interior of the associated hill region. At step 4, the local curvature at a peak with coordinates (x_i, y_j, f_{ij}) is calculated as the Gaussian curvature

$$C = \kappa_x \kappa_y = \frac{1}{\rho_x \rho_y},$$

where ρ_x is the radius of the circle interpolating the three points $(x_{i-1}, f_{i-1,j})$, $(x_i, f_{i,j})$ and $(x_{i+1}, f_{i+1,j})$, and ρ_y is the radius of the circle interpolating the three points $(y_{j-1}, f_{i,j-1})$, $(y_j, f_{i,j})$ and $(y_{j+1}, f_{i,j+1})$. It follows that

$$S_{\mathrm{pc}} = \frac{1}{n_{\mathrm{h}}} \sum_{i=1}^{n_{\mathrm{h}}} C_i,$$

where $C_i,\ i = 1, \ldots, n_{\mathrm{h}}$, are the local curvatures for the (significant) peaks.

C.3.5 Five-point peak height

The five-point peak height $S_{5\mathrm{p}}$ has the following designation [10, clause 6.8.3.1]:

$S_{5\mathrm{p}}$ = FC;H;Wolfprune:X%;Top:5;Lpvh;Mean

that is interpreted as follows:

1. Select hills (H);

2. Apply (hill) segmentation by Wolf pruning using a threshold of X % of S_{z} [6] and [10, clause 4.1.6];
3. Select the five hills with the greatest peak heights;
4. Set the attribute value for each selected hill to be the peak height;
5. Evaluate the arithmetic mean of the attribute values.

At step 2, the default threshold value for Wolf pruning is $5\ \%$. At step 3, only those hill regions that contain a peak strictly in the interior of the region are counted. Furthermore, if there are fewer than five peaks, no value for $S_{5\mathrm{p}}$ is returned. It follows that

$$S_{5\mathrm{p}} = \frac{1}{5}\sum_{i=1}^{5} P_i,$$

where $P_i,\ i = 1, \ldots, 5$, are the peak heights for the five hills with the greatest peak heights.

C.3.6 Five-point pit height

The five-point pit height $S_{5\mathrm{v}}$ has the following designation [10, clause 6.8.3.2]:

$S_{5\mathrm{v}}$ = FC;D;Wolfprune:X%;Bot:5;Lpvh;Mean

that is interpreted as follows:

1. Select dales (D);
2. Apply (dale) segmentation by Wolf pruning using a threshold of X % of S_{z} [6] and [10, clause 4.1.6];
3. Select the five dales with the greatest pit heights;
4. Set the attribute value for each selected dale to be the pit height;
5. Evaluate the arithmetic mean of the attribute values.

At step 2, the default threshold value for Wolf pruning is $5\ \%$. At step 3, only those dale regions that contain a pit strictly in the interior of the region are counted. Furthermore, if there are fewer than five pits, no value for $S_{5\mathrm{v}}$ is returned. It follows that

$$S_{5\mathrm{v}} = \frac{1}{5}\sum_{i=1}^{5} V_i,$$

where $V_i,\ i = 1, \ldots, 5$, are the pit heights for the five dales with the greatest pit heights.

C.3.7 Ten-point height of surface

The ten-point height S_{10z} is defined as the sum of S_{5p} and S_{5v} [10, clause 6.8.3]:

$$S_{10z} = S_{5p} + S_{5v}.$$

C.3.8 Mean dale area

The mean dale area $S_{da}(c)$ has the following designation [10, clause 6.8.4]:

$S_{da}(c)$ = FC;D;Wolfprune:X%;Open:c/Closed:c;AreaE;Mean

that is interpreted as follows:

1. Select dales (D);
2. Apply (dale) segmentation by Wolf pruning using a threshold of X % of S_z [6] and [10, clause 4.1.6];
3. Select the dale regions that are open at a height defined by the areal material ratio c (for the designation Open:c) or closed (for designation Closed:c);
4. Set the attribute value for each selected dale region to be the area of the region defined by that part of the dale below a height defined by the areal material ratio c;
5. Evaluate the arithmetic mean of the attribute values.

At step 2, the default threshold value for Wolf pruning is 5 %. At step 3, the default value for Open/Closed is closed. A dale region is selected if there exists a point (x_i, y_j) in the region with height $f_{ij} \leq S_{mc}(c)$. Furthermore, the selected dale region is counted as open if there exists a point (x_i, y_j) in the region that is on an edge of the evaluation area with height $f_{ij} < S_{mc}(c)$. Otherwise, the dale region is counted as closed. It follows that

$$S_{da}(c) = \frac{1}{n_d} \sum_{i=1}^{n_d} |A_i|,$$

where $|A_i|,\ i = 1, \ldots, n_d$, are the areas of the selected (open or closed) dale regions A_i.

C.3.9 Mean hill area

The mean hill area $S_{ha}(c)$ has the following designation [10, clause 6.8.5]:

$S_{ha}(c)$ = FC;H;Wolfprune:X%;Open:c/Closed:c;AreaE;Mean that is interpreted as follows:

1. Select hills (H);
2. Apply (hill) segmentation by Wolf pruning using a threshold of X % of S_z [6] and [10, clause 4.1.6];
3. Select the hill regions that are open at a height defined by the areal material ratio c (for the designation Open:c) or closed (for designation Closed:c);
4. Set the attribute value for each selected hill region to be the area of the region defined by that part of the hill above a height defined by the areal material ratio c;
5. Evaluate the arithmetic mean of the attribute values.

At step 2, the default threshold value for Wolf pruning is 5 %. At step 3, the default value for Open/Closed is closed. A hill region is selected if there exists a point (x_i, y_j) in the region with height $f_{ij} \geq S_\mathrm{mc}(c)$. Furthermore, the selected hill region is counted as open if there exists a point (x_i, y_j) in the region that is on an edge of the evaluation area with height $f_{ij} > S_\mathrm{mc}(c)$. Otherwise, the hill region is counted as closed. It follows that

$$S_\mathrm{ha}(c) = \frac{1}{n_\mathrm{h}} \sum_{i=1}^{n_\mathrm{h}} |A_i|,$$

where $|A_i|,\ i = 1, \ldots, n_\mathrm{h}$, are the areas of the selected (open or closed) hill regions A_i.

C.3.10 Mean dale volume

The mean dale volume $S_\mathrm{dv}(c)$ has the following designation [10, clause 6.8.6]:

$S_\mathrm{dv}(c)$ = FC;D;Wolfprune:X%;Open:c/Closed:c;VolE;Mean

that is interpreted as follows:

1. Select dales (D);
2. Apply (dale) segmentation by Wolf pruning using a threshold of X % of S_z [6] and [10, clause 4.1.6];
3. Select the dale regions that are open at a height defined by the areal material ratio c (for the designation Open:c) or closed (for designation Closed:c);
4. Set the attribute value for each selected dale region to be the volume of the dale below a height defined by the areal material ratio c;

5. Evaluate the arithmetic mean of the attribute values.

At step 2, the default threshold value for Wolf pruning is 5 %. At step 3, the default value for Open/Closed is closed. A dale region is selected if there exists a point (x_i, y_j) in the region with height $f_{ij} \leq S_{\mathrm{mc}}(c)$. Furthermore, the selected dale region is counted as open if there exists a point (x_i, y_j) in the region that is on an edge of the evaluation area with height $f_{ij} < S_{\mathrm{mc}}(c)$. Otherwise, the dale region is counted as closed. It follows that

$$S_{\mathrm{dv}}(c) = \frac{1}{n_{\mathrm{d}}} \sum_{i=1}^{n_{\mathrm{d}}} |V_i|,$$

where $|V_i|,\ i = 1, \ldots, n_{\mathrm{d}}$, are the volumes defined by the selected (open or closed) dale regions.

C.3.11 Mean hill volume

The mean hill volume $S_{\mathrm{hv}}(c)$ has the following designation [10, clause 6.8.7]:

$S_{\mathrm{hv}}(c)$ = FC;H;Wolfprune:X%;Open:c/Closed:c;VolE;Mean

that is interpreted as follows:

1. Select hills (H);
2. Apply (hill) segmentation by Wolf pruning using a threshold of X % of S_{z} [6] and [10, clause 4.1.6];
3. Select the hill regions that are open at a height defined by the areal material ratio c (for the designation Open:c) or closed (for designation Closed:c);
4. Set the attribute value for each selected hill region to be the volume of the hill above a height defined by the areal material ratio c;
5. Evaluate the arithmetic mean of the attribute values.

At step 2, the default threshold value for Wolf pruning is 5 %. At step 3, the default value for Open/Closed is closed. A hill region is selected if there exists a point (x_i, y_j) in the region with height $f_{ij} \geq S_{\mathrm{mc}}(c)$. Furthermore, the selected hill region is counted as open if there exists a point (x_i, y_j) in the region that is on an edge of the evaluation area with height $f_{ij} > S_{\mathrm{mc}}(c)$. Otherwise, the hill region is counted as closed. It follows that

$$S_{\mathrm{hv}}(c) = \frac{1}{n_{\mathrm{h}}} \sum_{i=1}^{n_{\mathrm{h}}} |V_i|,$$

where $|V_i|,\ i = 1, \ldots, n_{\mathrm{h}}$, are the volumes defined by the selected (open or closed) hill regions.