Uncertainty evaluation for the calculation of a surface texture parameter in the profile case

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Abstract

Consideration is given to the problem of uncertainty evaluation for an amplitude surface texture parameter. A formulation of the problem is given in terms of a measurement model that defines the mathematical relationship between all quantities involved in the measurement and the available information about those quantities. The applications of two approaches to solving the formulated problem, namely the conventional approach of 'uncertainty propagation' and a Monte Carlo method that provides a more generally applicable approach, are indicated. Results obtained from the two approaches for simulated measurement data are compared. It is found that, for a particular amplitude parameter, there can be appreciable differences between the estimates of the parameter although the standard uncertainties returned by the two methods are in good agreement.

Keywords: surface texture parameter, uncertainty evaluation, GUM uncertainty framework, Monte Carlo method

1. Introduction

Knowledge of the topography of a machined surface is necessary in order to understand the functional performance of the surface, and is consequently essential to the manufacturing process. Surface texture parameters defined in ISO specification standards are widely used to assign numerical values to the measured topography. However, if the values of a parameter are to be used for the comparison of different surfaces and for the interpretation of surface texture tolerances on engineering drawings, it is helpful if statements of uncertainty accompany the values. Furthermore, it is important that those statements are reliable.

The problem of uncertainty evaluation for a particular amplitude surface texture parameter, viz. the root mean square value of a surface profile, is considered. A formulation of the problem is given in terms of a measurement model that defines the mathematical relationship between all quantities involved in the measurement and the available information about those quantities. The applications of two approaches to solving the formulated problem, namely the conventional approach of the GUM uncertainty framework and a Monte Carlo method that provides a more generally applicable approach, are indicated. Results obtained from the two approaches for simulated measurement data are compared.

The procedure for evaluating a surface texture parameter is a complicated one, involving a number of steps, including the application of filters. Furthermore, the relevant specification standards that define these steps do so

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in terms of continuous (analogue) representations of the profile, and the definitions need to be interpreted appropriately if they are to be applied to the discrete (digital) representations that are available in practice. The definitions, which are typically presented as integrals of functions of the continuous profile, are formulated in this work as quadrature rules involving the measured coordinate values, and expressed as discrete summations involving those values. This formulation allows for the consideration of uncertainties associated with both (abscissa and ordinate) coordinate values of the measured points. A treatment of this general case is available [1]. This paper summarizes that treatment for the particular case that there are uncertainties associated with the ordinate values only. For some related work, see, e.g., [2, 3, 4, 5].

For the purpose of controlling a production process, many users will want to include the influence of (the variability of) the surface on the surface texture parameter. Consideration is not given to this aspect here. Instead, the concern is with quantifying the influence of the measuring system only on the parameter.

2. Measurement uncertainty evaluation

The basis for the evaluation of measurement uncertainty is the propagation of probability distributions. In order to apply the propagation of distributions, a measurement model of the generic form $Y = f(X_1, \ldots, X_N)$ relating input quantities $X_1, \ldots, X_N$, about which information is available, and the output quantity Y, about which information is required, is formulated. An implementation of the propagation of distributions provides a probability distribution for Y, from which can be obtained an estimate of Y, the standard uncertainty associated with the estimate, and a coverage interval for Y corresponding to a stipulated coverage probability. Particular implementations of the approach are the GUM uncertainty framework [6] and a Monte Carlo method [7].

The primary guide in metrology on uncertainty evaluation is the 'Guide to the expression of uncertainty in measurement' (GUM) [6], which presents a framework for uncertainty evaluation based on the use of the law of propagation of uncertainty (LPU) and the central limit theorem. For independent X_i, LPU is used to provide the estimate $y = f(x_1, \ldots, x_N)$ of Y with associated standard uncertainty $u(y) = \Sigma_i\, c_i^2 u^2(x_i)$, in which x_i is an estimate of X_i with associated standard uncertainty $u(x_i)$, and c_i is the ith sensitivity coefficient equal to the derivative of f with respect to X_i evaluated at the estimates x_i. The formula for $u(y)$ is modified to account for the covariances $\mathrm{cov}(x_i, x_j)$ associated with x_i and x_j when the X_i are mutually dependent.

A Monte Carlo method for uncertainty evaluation provides an implementation of the propagation of distributions that is free from the limitations and approximations inherent in the GUM uncertainty framework, such as linearization of the measurement function f in the application of LPU. The method operates in the following manner. A random draw is made from the joint probability distribution for the input quantities and the corresponding value of Y is formed by evaluating the measurement function for these values. Many Monte Carlo trials are performed, i.e., the process is repeated many times, to obtain M, say, values y_k, $k = 1, \ldots, M$, of Y. The values y_k, $k = 1, \ldots, M$, provide a discrete representation of the probability distribution for Y from which are determined an estimate of Y as the expectation, the standard uncertainty associated with the estimate as the standard deviation, and a coverage interval for Y corresponding to a stipulated coverage probability.

Multivariate measurement models of the generic form $\boldsymbol{Y} = \boldsymbol{f}(\boldsymbol{X})$, which relate input quantities $\boldsymbol{X} = (X_1, \ldots, X_N)^{\mathrm{T}}$ and output quantities $\boldsymbol{Y} = (Y_1, \ldots, Y_M)^{\mathrm{T}}$ through functions $\boldsymbol{f} = (f_1, \ldots, f_M)^{\mathrm{T}}$, are an important class of model. The output quantities are generally mutually correlated because they depend on common input quantities. Generalizations of the GUM uncertainty framework and a Monte Carlo method for uncertainty evaluation apply to such multivariate measurement models [8].

3. Measurement models

The following steps are undertaken to calculate surface texture parameters for a measured surface profile $z(x)$: (1) remove geometric form, (2) mean centre the resulting profile, (3) apply filters to obtain primary, waviness and roughness profiles, and (4) evaluate surface texture parameters for the primary, waviness and roughness profiles. A specification of these steps is used to establish measurement models of the generic forms described in section 2 in terms of which an estimate of the surface texture parameter and the associated uncertainty are determined. In practice, knowledge of $z(x)$ takes the form of measured points (x_i, z_i), $i = 1, \ldots, n$, with the values x_i having a uniform spacing h, and the above steps are undertaken in terms of such (discrete) data.

Because different surface geometries may be measured, including planes, spheres and cylinders, and the approach to removing form can depend on the particular geometry, it is assumed that step (1) has already been performed. Step (2), which is not well defined in specification standards, consists of evaluating ordinate values y_i, $i = 1, \ldots, n$, given by

$$y_i = z_i - \sum_{j=1}^{n} z_j / n, \tag{1}$$

where $y_i = y(x_i)$ and $y(x)$ is the mean-centred profile. Other interpretations of this step, e.g., expressed in terms of the continuous profile $z(x)$ or accounting for the uncertainties associated with the measured values z_i, could also be considered.

In step (3), the primary profile $p(x)$ is derived from $y(x)$ by filtering $y(x)$ to remove (very) short-wavelength components associated with measurement noise using a filter with cut-off wavelength λs.[1] The waviness profile $w(x)$ and roughness profile $r(x)$, satisfying $p(x) = w(x) + r(x)$, are then derived from $p(x)$ using a filter with cut-off wavelength λc, where $\lambda\text{c} > \lambda\text{s}$. For example, $p(x)$ is defined by points with coordinates (x_i, p_i), where p_i is evaluated from the discrete convolution

$$p_i = h \sum_{j=-m}^{m} s(-jh, \lambda\text{s}) y_{i+j}, \quad s(x, \lambda) = \frac{1}{\alpha\lambda} \exp\left[-\pi \left(\frac{x}{\alpha\lambda} \right)^2 \right], \quad \alpha = \sqrt{\log 2/\pi}. \tag{2}$$

With m chosen large enough so that $s(-mh) = s(mh) = 0$, formula (2) can be derived by applying the trapezoidal quadrature rule to the definition of $p(x)$ as a continuous convolution [9]. The formula, which is an approximation to the continuous convolution, is an interpretation of the definition in terms of the available discrete data.

The example of a surface texture parameter considered here is the root mean square value, denoted by Pq, Wq and Rq according to whether the evaluated profile is $p(x)$, $w(x)$ or $r(x)$. For example, Pq is evaluated in step (4) from

$$Pq = \sqrt{\frac{h}{L} \sum_i{}'' p_i^2}, \tag{3}$$

where L is the length along the x-coordinate axis of the profile, and the superscript on the summation indicates that the first and last terms are multiplied by one half. Again, formula (3) is derived by applying the trapezoidal quadrature rule to the definition of Pq in terms of the continuous profile $p(x)$ [9]. Comparable formulae apply for Wq and Rq.

4. Measurement data

The measurement data consists of estimates (x_i, z_i) of quantities (X_i, Z_i) representing the coordinates of points defining the measured profile. Consider uncertainties associated with the ordinate values only, described by the observation model $X_i = x_i$ and $Z_i = z_i + E_0 + E_i$, in which E_0 and E_i are independent random variables with expectations zero and variances $V(E_0) \equiv u_s^2 = \rho\, u_z^2$ and $V(E_i) \equiv u_r^2 = (1 - \rho)\, u_z^2$. Here, E_0 can be considered as representing a systematic error quantity common to the measured quantities Z_i, and the E_i as random error

[1] The notation λs and λc for cut-off wavelength, and Pq, Wq and Rq for surface texture parameter, is that used in the relevant specification standard.

quantities different for each Z_i. Values for ρ, the correlation coefficient associated with values z_i and z_j, and u_z, the standard uncertainty associated with the values z_i, are obtained from knowledge of the instrument used to make the measurements. For example, NanoSurf IV is a stylus instrument designed at NPL to provide traceability in the measurement of surface texture. There are four dominant sources of uncertainty, corresponding to the metrology frame, imperfections in the optics, deadpath length, and Abbe error. Indicative values for the standard uncertainties associated with the four effects are available [11]. Although the observation model considered is quite simplistic, it describes adequately many measurement situations. Consideration of more complicated models is nevertheless also possible.

5. Results

The points defining a measured profile are simulated as following:

$$x_i = ih, \quad z_i = a_1 \sin(2\pi x_i / L_1) + a_2 \sin(2\pi x_i / L_2) + e_0 + e_i, \quad i = 1, \ldots, n,$$

where h = 0.5 µm, $n = 2m_s + 7m_c$ with $m_s = \lambda s/h$ and $m_c = \lambda c/h$, $a_1 = a_2$ = 10 nm, L_1 = 400 µm, L_2 = 30 µm, λs = 2.5 µm, λc = 80 µm, and e_0 and e_i are random draws made independently from, respectively, $N(0, \rho\, u_z^2)$ and $N(0, (1 - \rho)\, u_z^2)$ with u_z = 1 nm. Consider two problems: (i) ρ = 0, i.e., the measured ordinate values have no associated correlation, and (ii) ρ = 0.9, i.e., the values are influenced by a (dominant) systematic effect.

Fig. 1 illustrates the approximations to the probability distributions for Pq, Wq and Rq obtained using the GUM uncertainty framework and a Monte Carlo method. For the former, the approximation takes the form of a Gaussian (normal) distribution with expectation and standard deviation obtained using LPU. For the latter, the approximation takes the form of a scaled frequency distribution (histogram). In each case, the origin (zero value of the quantity) corresponds to the estimate (pq, wq or rq, respectively) provided by LPU.

Results for profiles simulated to have a range of characteristics, as well as those simulating measurements made by NPL's NanoSurf IV instrument, are available [1].

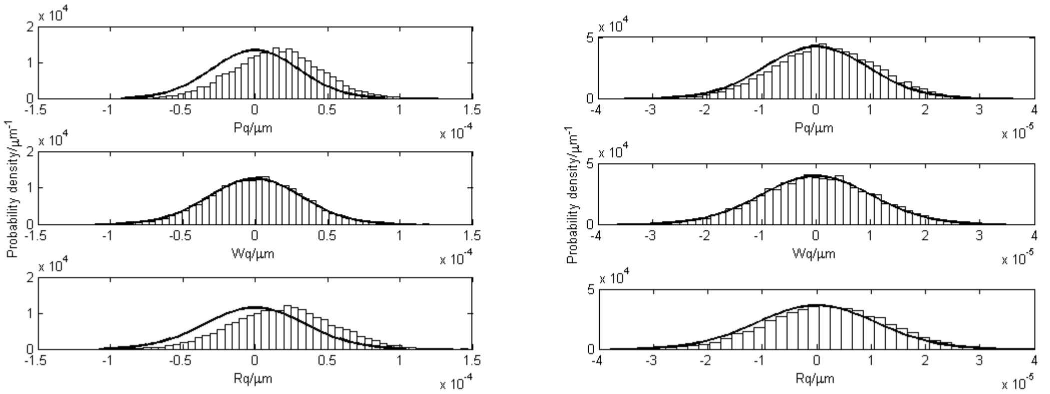

Fig. 1. Approximations to the probability distributions for the quantities $Pq - pq$ (top), $Wq - wq$ (middle) and $Rq - rq$ (bottom) for the problems ρ = 0 (left) and ρ = 0.9 (right).

6. Conclusions

For the particular surface texture parameter considered, the results indicated in section 5 and presented more fully in [1], suggest that there is generally good agreement between the standard uncertainties provided by the

GUM uncertainty framework and those provided by a Monte Carlo method. However, there are cases when the estimates of the parameter for the primary and roughness profiles returned by the two approaches can be appreciably different, notably when the uncertainties associated with the measured coordinates are large compared with the amplitude of the profile. For the results given in section 5, the difference is most obvious for the case of ordinate values with no associated correlation, i.e., $\rho = 0$.

Furthermore, the uncertainty is strongly influenced by the (component of) uncertainty associated with the measured ordinate values that quantifies (independent) random errors. The operation of mean centering the measured profile (and, to a lesser extent, of filtering the profile) has the effect of removing contributions made by (common) systematic error quantities to the measured coordinates.

Since the measurement functions defining the steps of mean centering and filtering are linear, the results returned by LPU of the GUM uncertainty framework are exact for these steps. However, the step of evaluating the surface texture parameter considered in terms of the evaluated (filtered) profiles is non-linear, for which LPU can be expected to be approximate. The degree of approximation can depend on properties of the evaluated profile, such as the magnitude of the uncertainties associated with the measured coordinates.

Software implementation of the law of propagation uncertainty requires storing and manipulating large matrices of data. For the simulations considered in section 5, the number of measured points is $n = 1\,130$. The covariance matrix associated with the coordinates defining the measured profile is a matrix of dimension $1\,130 \times 1\,130$ when uncertainties are associated with the ordinate values (and dimension $4\,520 \times 4520$ when uncertainties are associated with both coordinates). Consideration should be given to exploiting the structure of covariance and sensitivity matrices involved in the calculations. For example, covariance matrices are symmetric, and those encountered in this work can often by summarized (perhaps approximately) by an autocovariance function. In contrast, a Monte Carlo method, which is a generally applicable procedure, requires only to undertake (albeit many times for different values of the measured data) the evaluation of the surface texture parameter, and efficient implementations of the method are available [12].

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