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Uncertainty in the TBLC and bulk dissipation corrections for TCU1 / Argon

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UNCERTAINTY IN THE TBLC AND BULK DISSIPATION CORRECTIONS FOR TCU1 / ARGON

Gavin Sutton
Engineering Measurement

ABSTRACT

The uncertainty in the calculated Thermal Boundary Layer Correction (TBLC) / Bulk dissipation correction due to the uncertainty in the input parameters is calculated for the sphere TCU1 and argon gas. For typical measurement uncertainties, we find that the combined fractional uncertainty is a maximum at the lowest measurement pressure (50 kPa) and varies from:

0.26 PPM to 0.10 PPM for *dF/F* moving from the (0,2) to (0,10) mode.
0.28 PPM to 0.17 PPM for *G/F* moving from the (0,2) to (0,10) mode.

At P = 100 kPa, both uncertainties are less than 0.2 PPM.

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Approved on behalf of the Managing Director, NPL
by Martyn Sene, Director of Operations.

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NOMINAL PARAMETERS:

T = 273.16 K
a = 0.050 m
m = 0.039947706 kg.mol^{-1}
h = 1 (thermal acc. coef.)

The uncertainty in the calculated TBLC / Bulk dissipation correction due to the uncertainty in the input parameters is considered. Table 1 shows the typical uncertainties used in this study. Other parameters such as *density*, C_p, C_v, γ (C_p/C_v), β(2nd acoustic virial coefficient) and δ(3rd acoustic virial coefficient) are derived from these parameters and are not considered individually.

Item	Parameter	Description	Parameter uncertainty
1	T	Temperature	50 mK
2	P	Pressure	10 Pa
3	m	Molar mass	0.1%
4	a	Radius	0.1%
5	λ	Thermal conductivity	0.1%
6	η	Viscosity	0.1%
7	B	2nd virial coefficient	1.0%
8	dB/dT	1st derivative of 2nd virial coefficient w.r.t T	1.0%
9	d^2B/dT^2	2st derivative of 2nd virial coefficient w.r.t. T	1.0%
10	C	3rd virial coefficient	10.0%
11	dC/dT	1st derivative of 2nd virial coefficient w.r.t T	10.0%
12	d^2C/dT^2	2st derivative of 2nd virial coefficient w.r.t. T	10.0%

Table 1 input parameter uncertainties used to determine the uncertainty in the TBLC / Bulk dissipation correction.

PROCEDURE

- The TBLC/Bulk dissipation correction is first calculated for the nominal parameters to give a baseline value. Radial acoustic modes (0,2) to (0,10) are considered.
- The value of each parameter, in turn, is increased by its estimated uncertainty (last column in Table 1) and the TBLC/Bulk dissipation correction recalculated.
- The difference between the two values is plotted against pressure from 10 kPa to 610 kPa and indicates the uncertainty of the correction due to the uncertainty in the input parameter.
- Finally, the uncertainty contributions are summed in quadrature to give an overall uncertainty in the TBLC/Bulk dissipation correction.

NOMINAL CORRECTION

Figures 1 and 2 show the TBLC/Bulk dissipation correction for the frequency, F and the halfwidth, G respectively. The correction reduces as the mode number increases and the pressure increases.

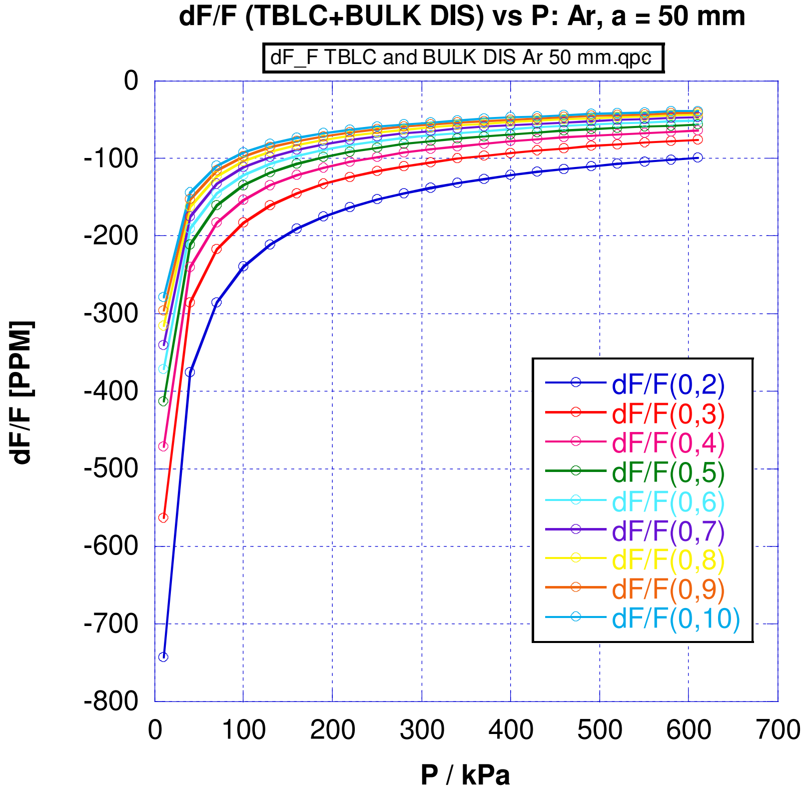

Figure 1 TBLC/Bulk dissipation correction: dF/F (baseline).

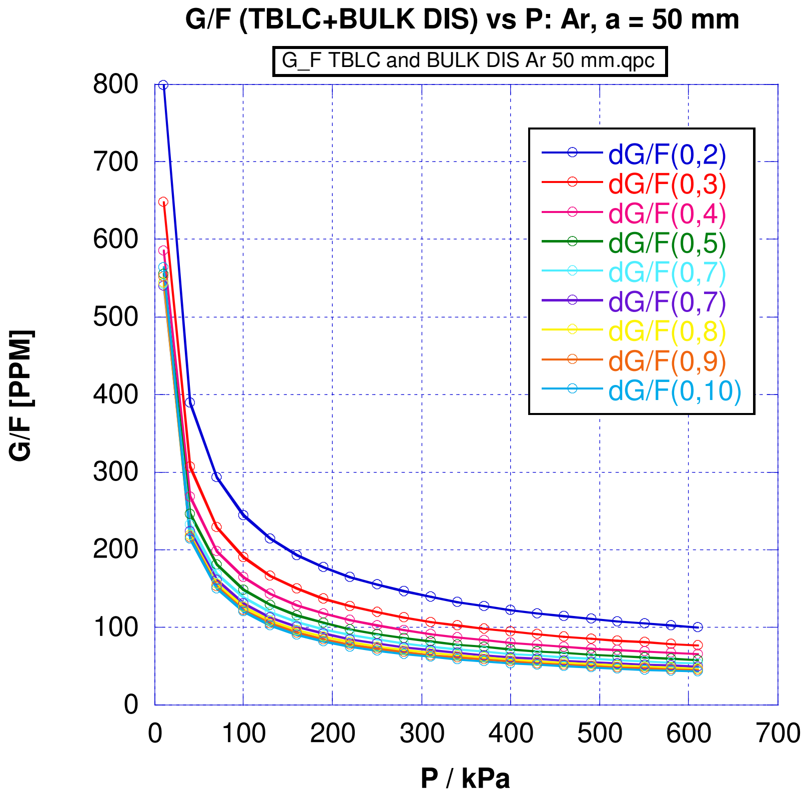

Figure 2 TBLC/Bulk dissipation correction: G/F (baseline).

1. U[TBLC/BULK DIS] DUE TO *U(T)* = 50 MK

With the TBLC / Bulk dissipation correction calculated at the measurement temperature, we expect *U(T)* < 1 mK. We chose a worst-case scenario in which *U(T)* = 50 mK but the uncertainty in the correction is still very small.

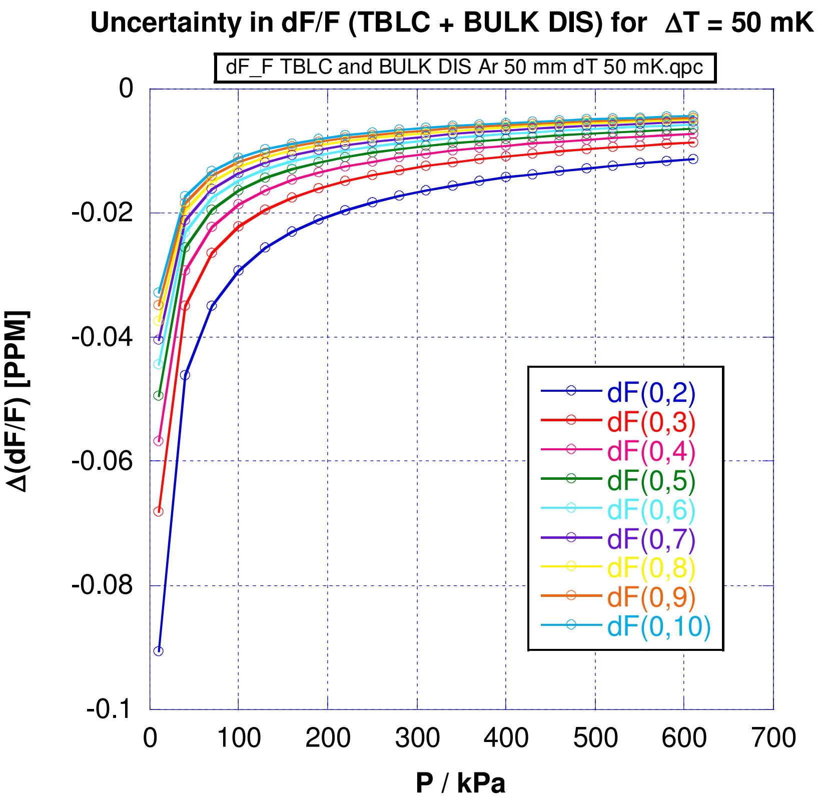

Figure 3

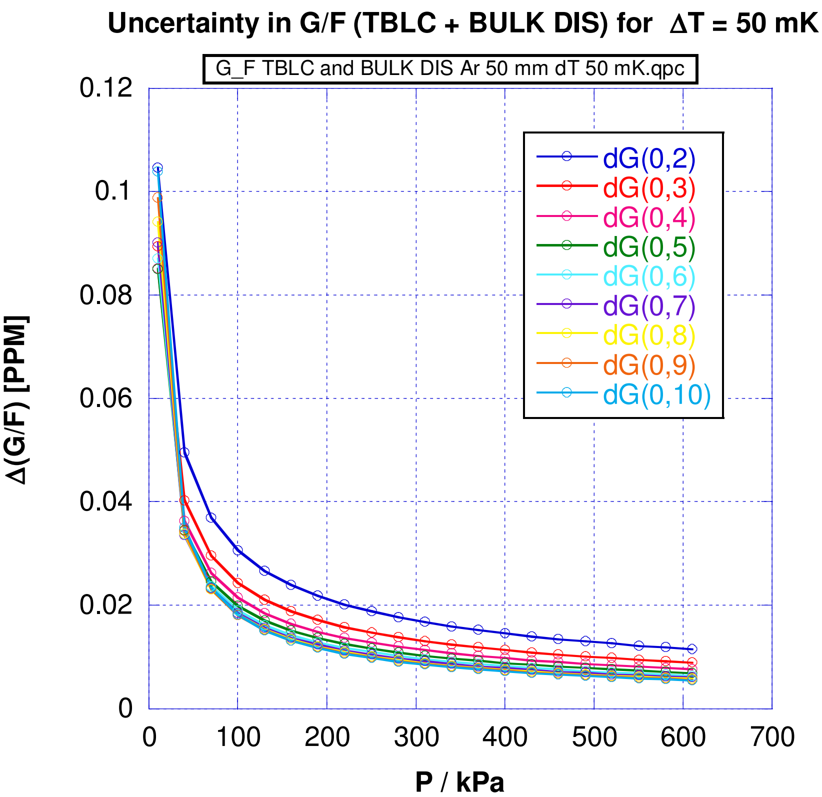

Figure 4

2. U[TBLC/BULK DIS] DUE TO *U(P)* = 10 Pa

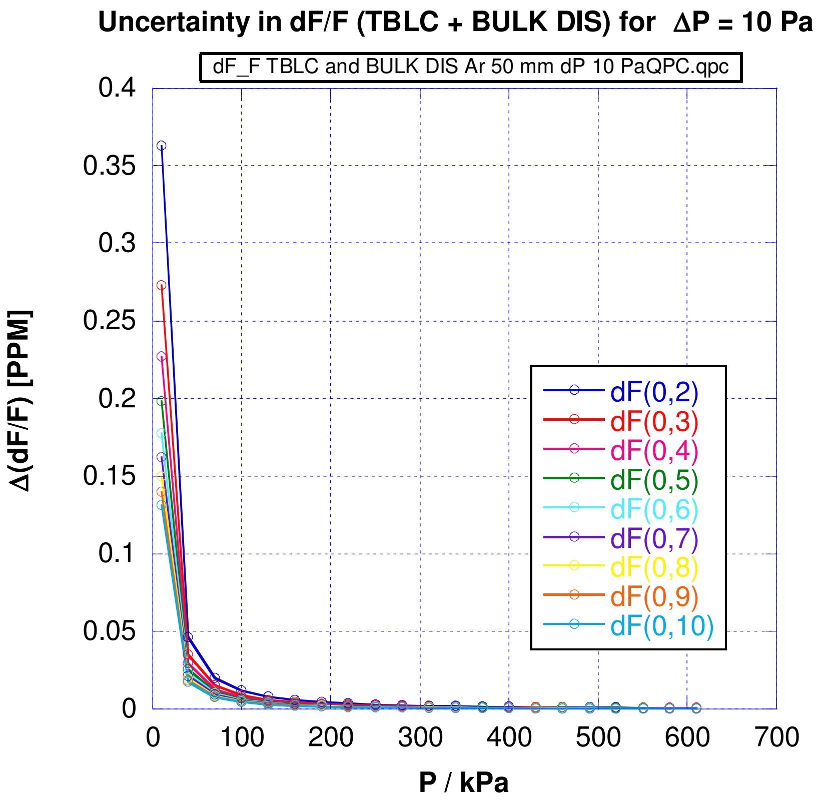

Figure 5

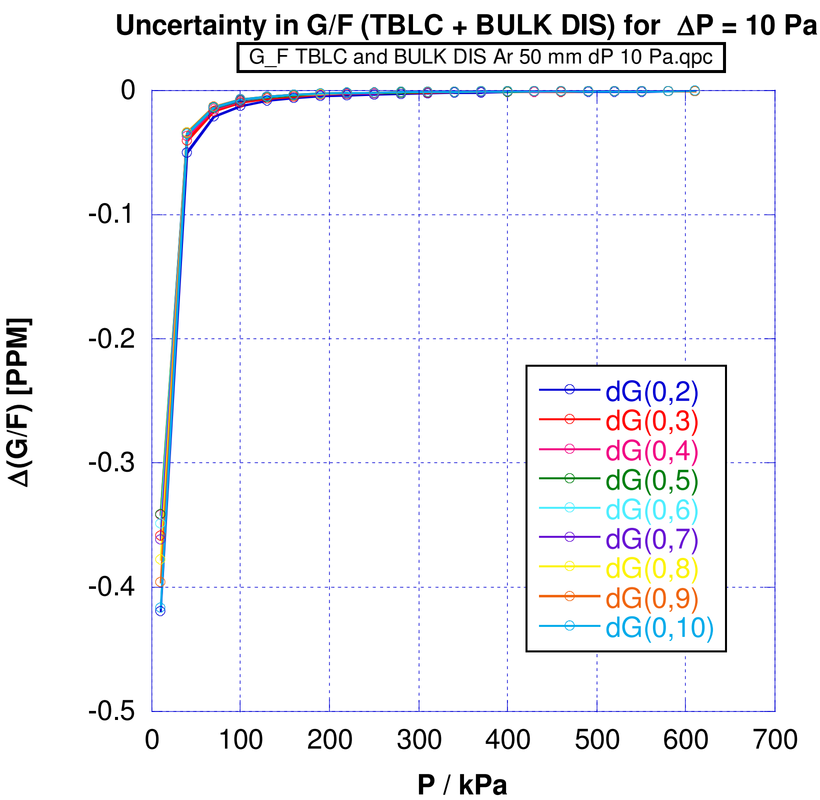

Figure 6

3. U[TBLC/BULK DIS] DUE TO $U(m)/m = 0.1\%$

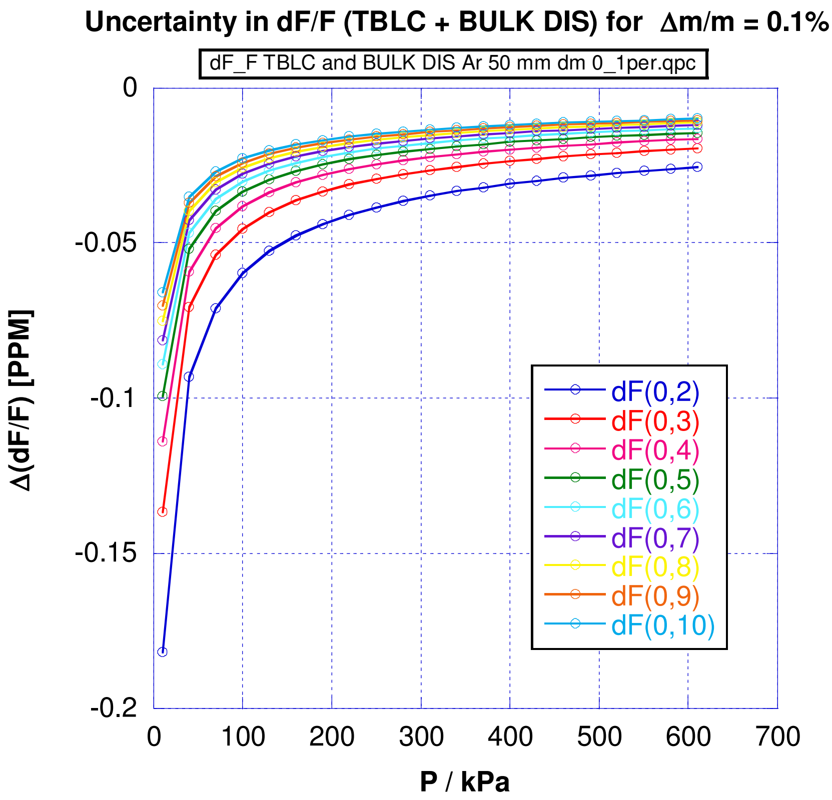

Figure 7

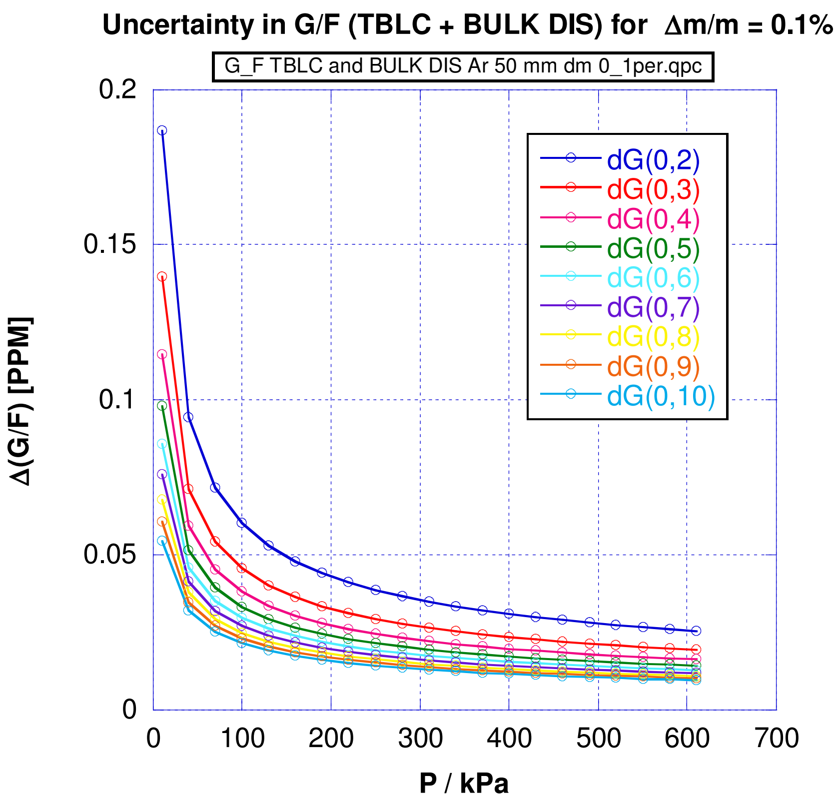

Figure 8

4. U[TBLC/BULK DIS] DUE TO $U(a)/a = 0.1\%$

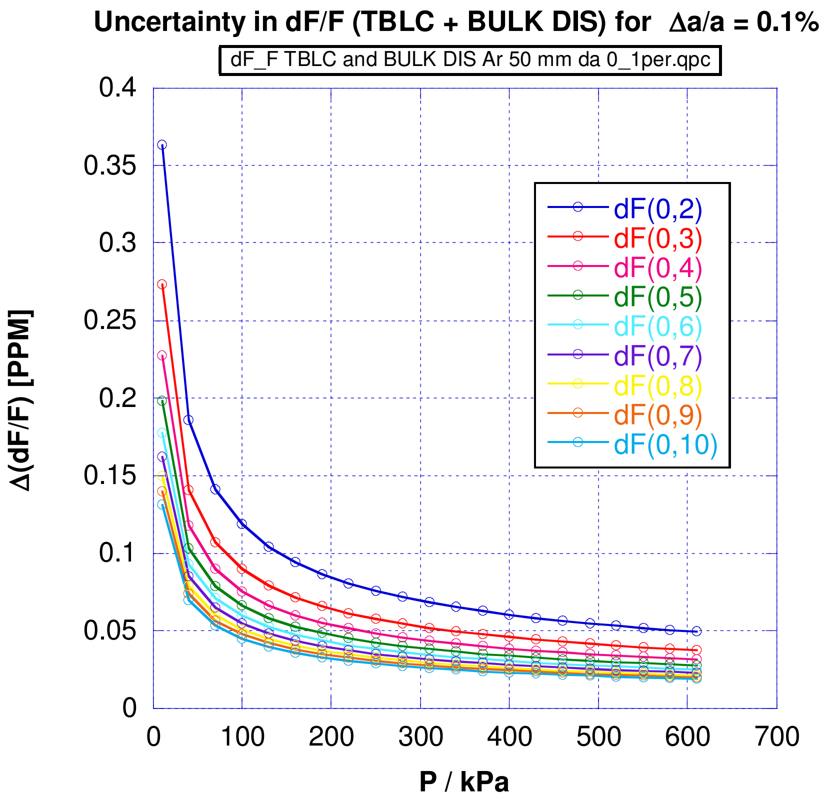

Figure 9

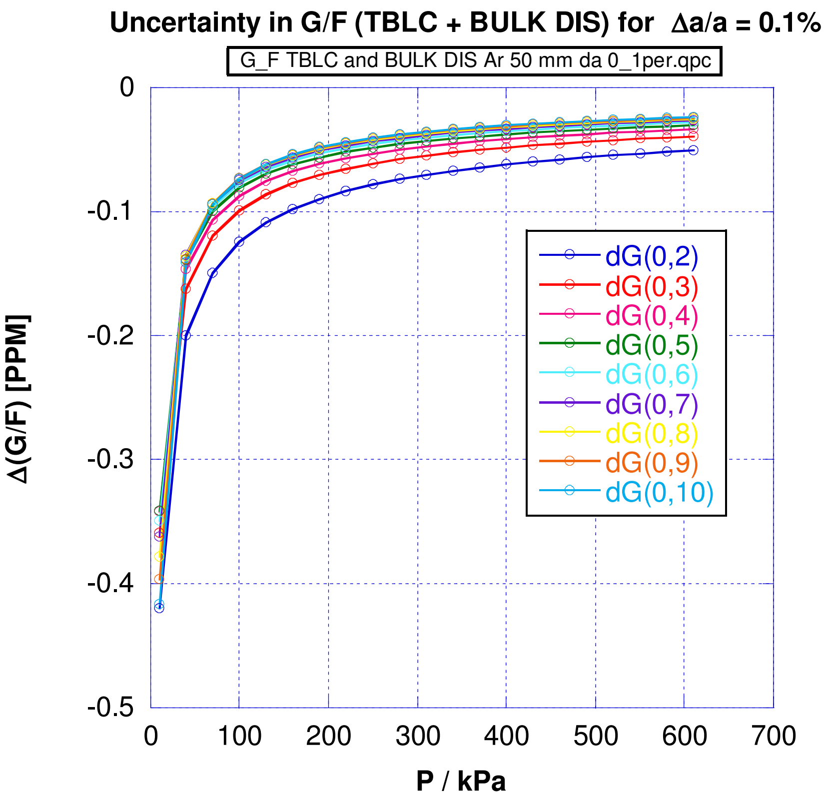

Figure 10

5. U[TBLC/BULK DIS] DUE TO $U(\lambda)/\lambda = 0.1\%$

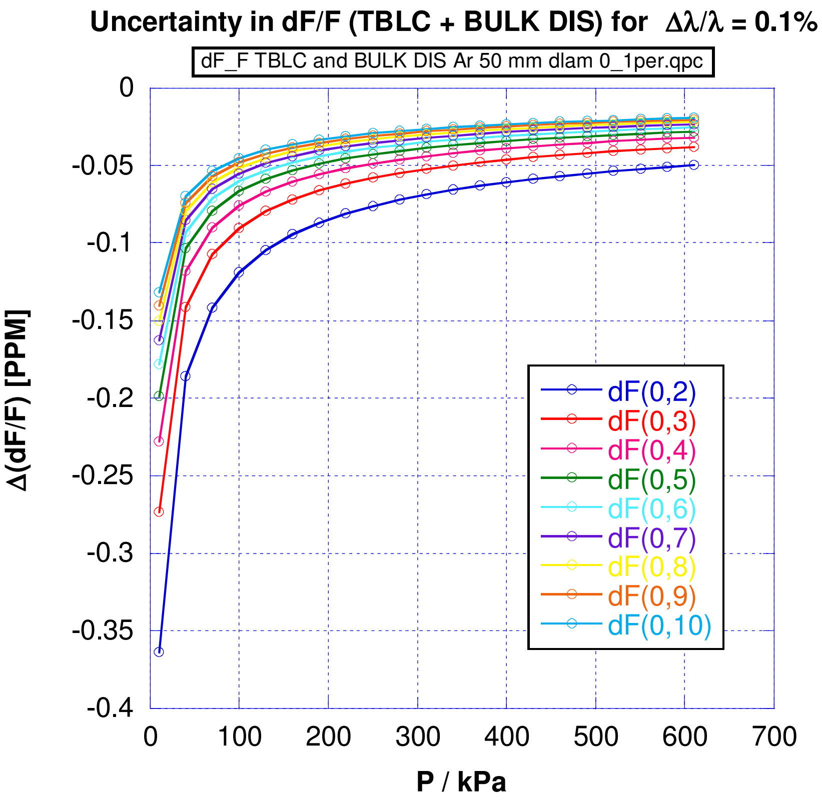

Figure 11

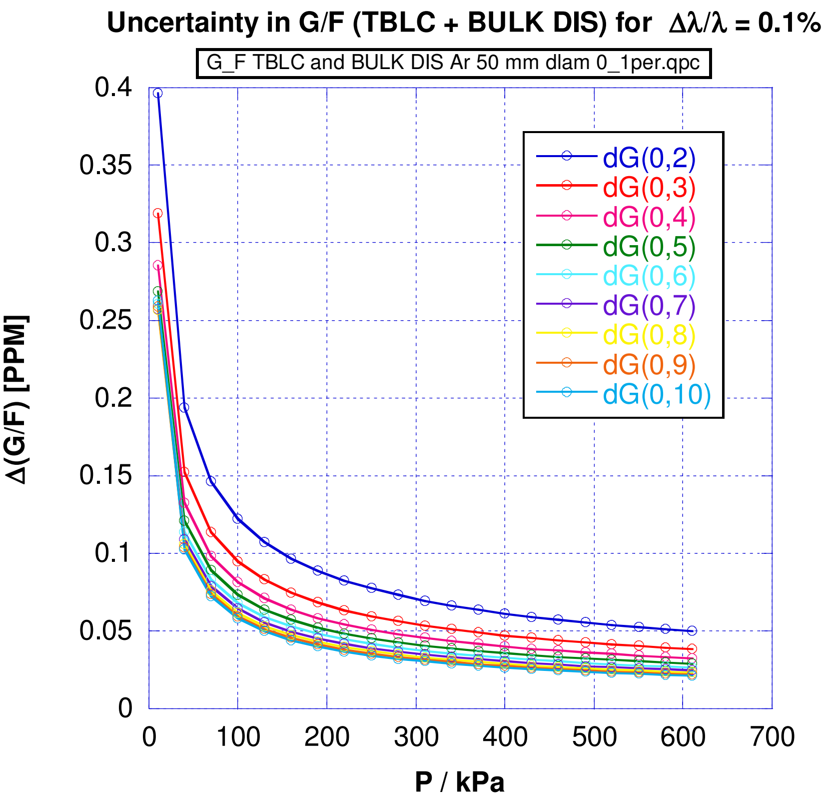

Figure 12

6. U[TBLC/BULK DIS] DUE TO $U(\eta)/\eta = 0.1\%$

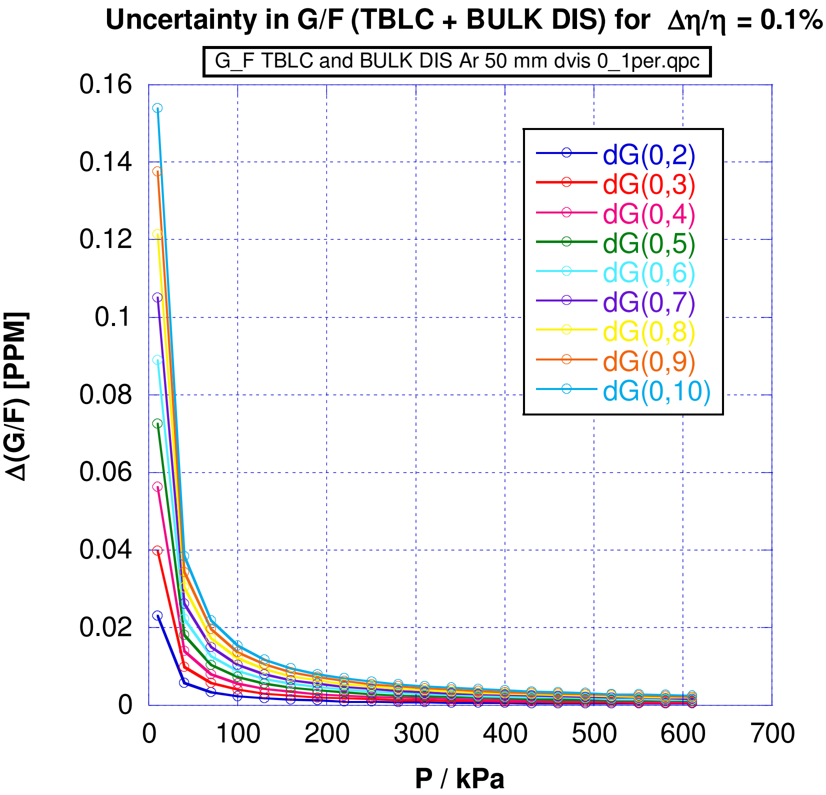

Figure 13

7. U[TBLC/BULK DIS] DUE TO $U(B)/B = 1.0\%$

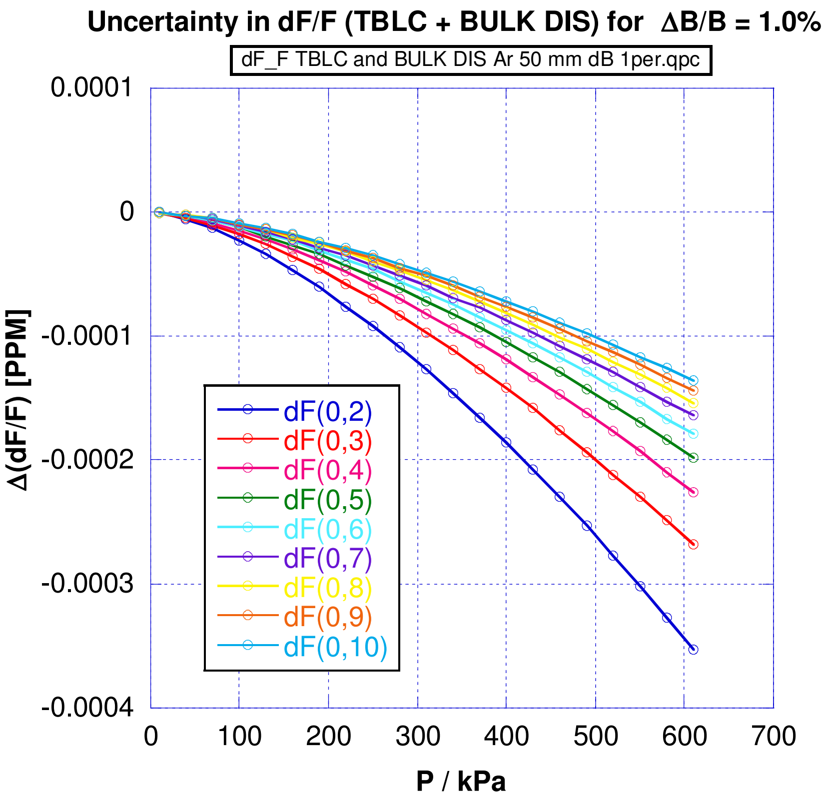

Figure 14

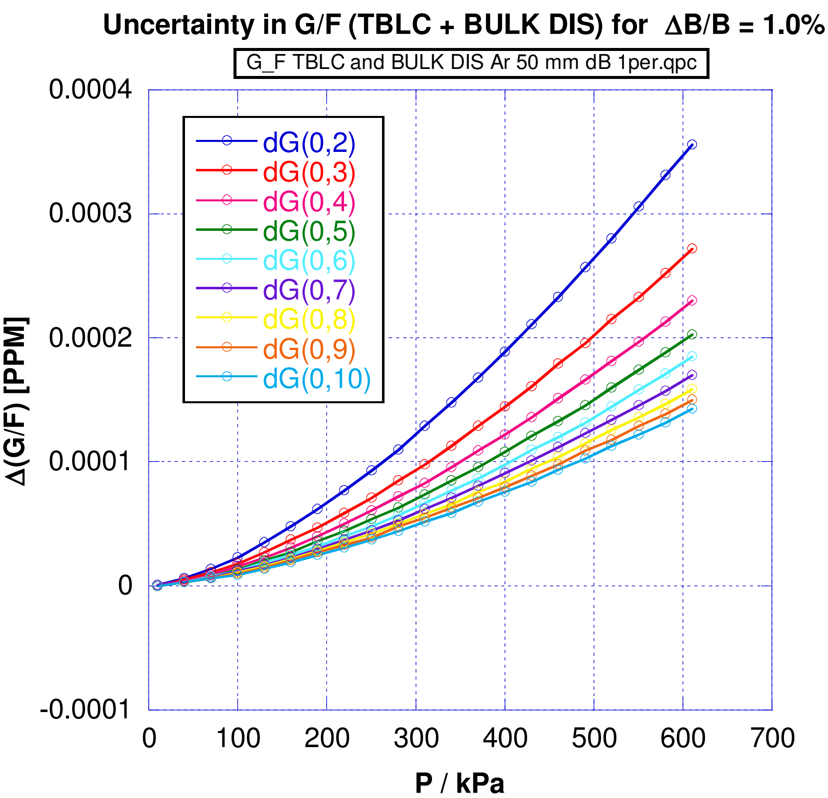

Figure 15

8. U[TBLC/BULK DIS] DUE TO *U(dB/dT)/ dB/dT* = 1.0%

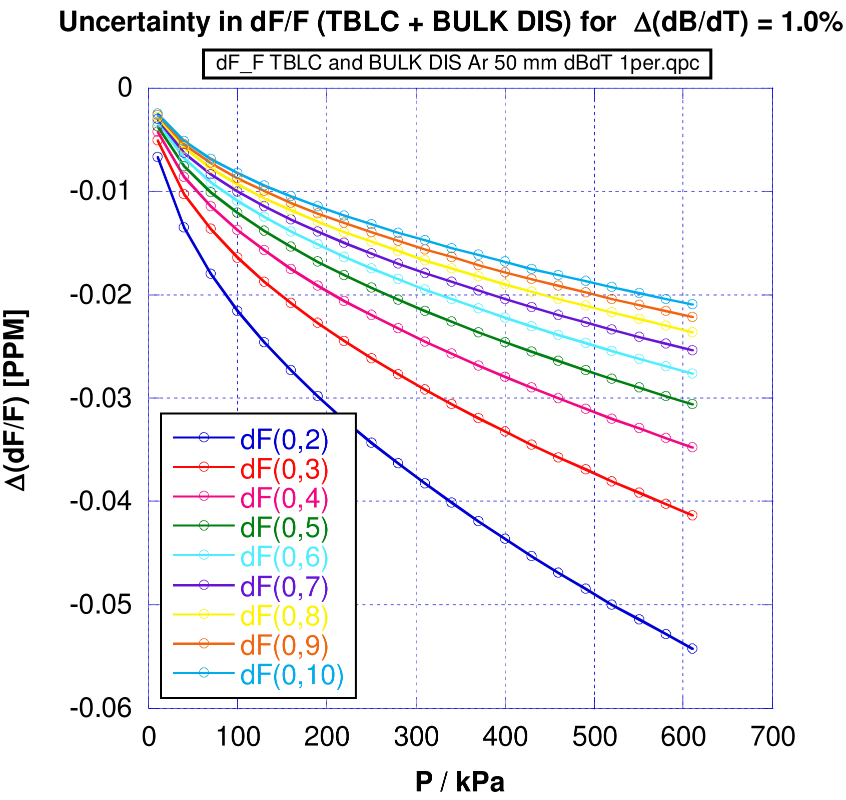

Figure 16

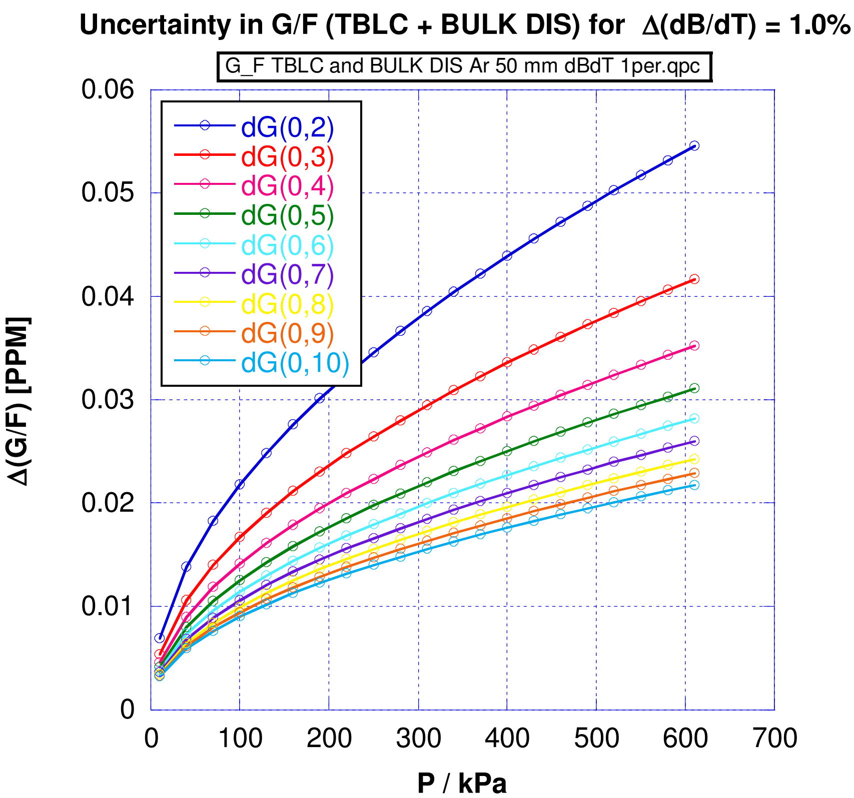

Figure 17

9. U[TBLC/BULK DIS] DUE TO $U(d^2B/dT^2)/\ d^2B/dT^2 = 1.0\%$

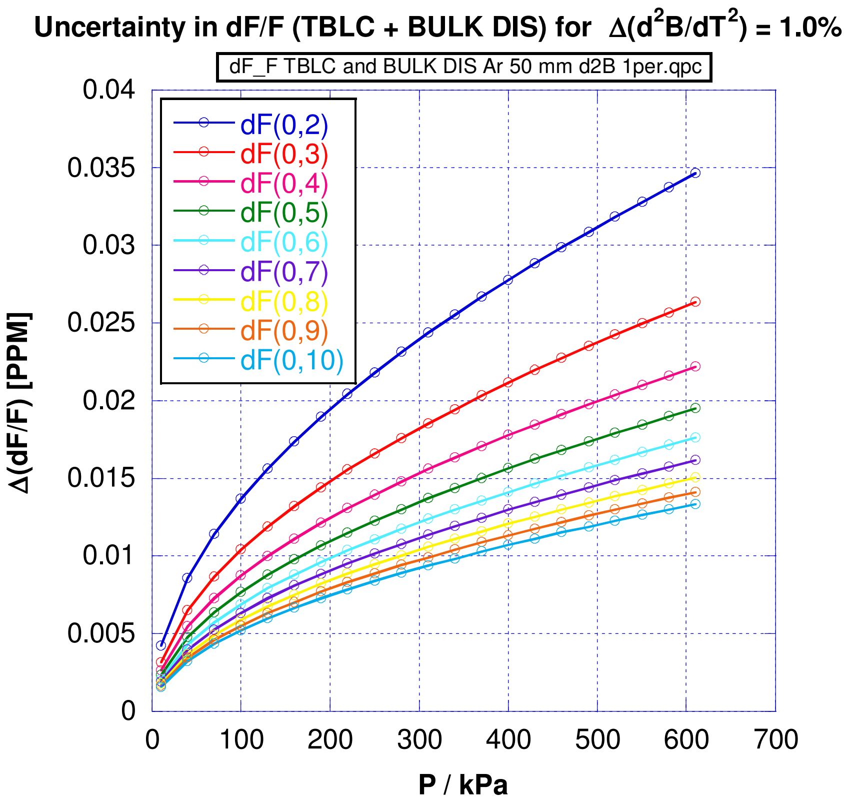

Figure 18

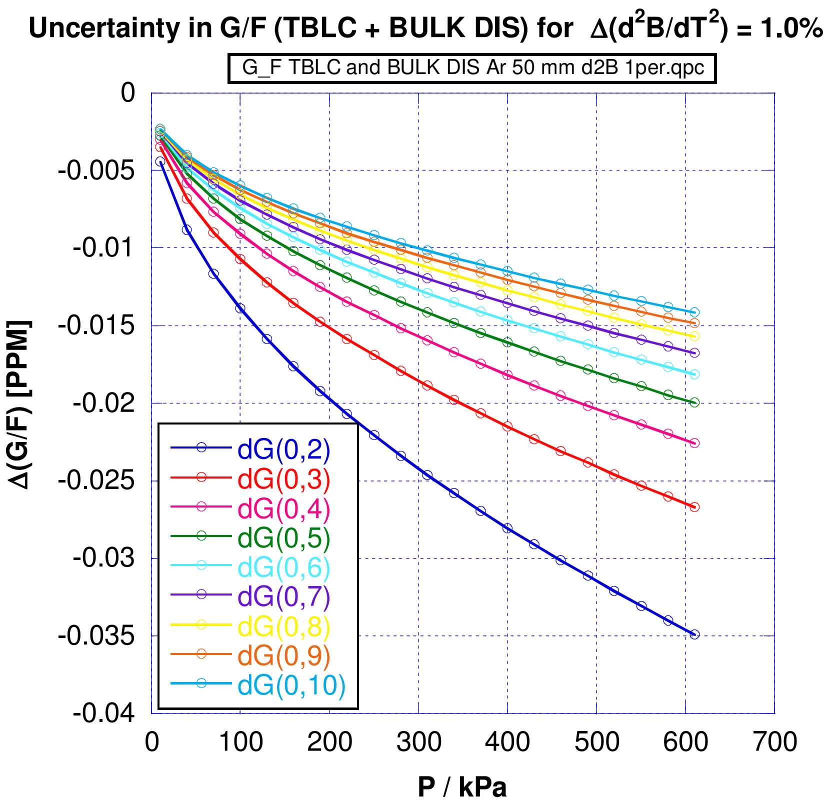

Figure 19

10. U[TBLC/BULK DIS] DUE TO *U(C)/C* = 10%

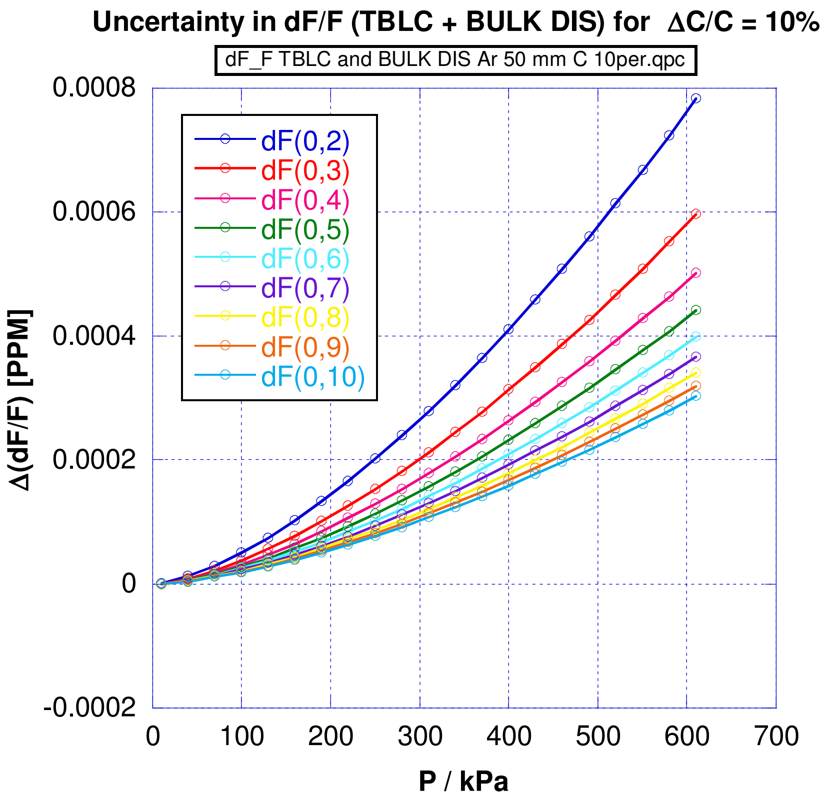

Figure 20

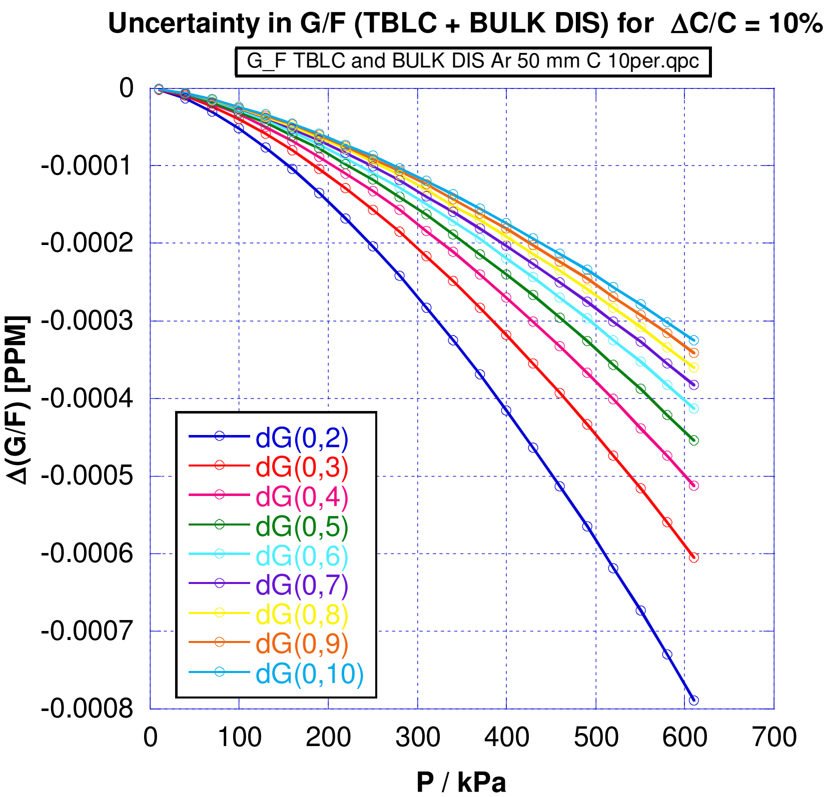

Figure 21

11. U[TBLC/BULK DIS] DUE TO *U(dC/dT)/dC/dT* = 10%

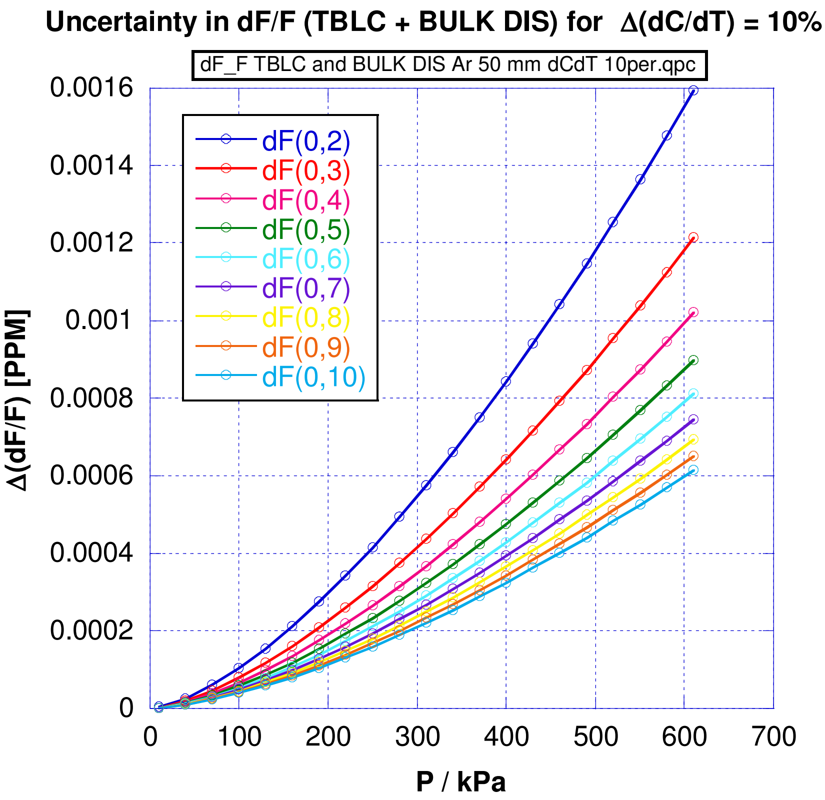

Figure 22

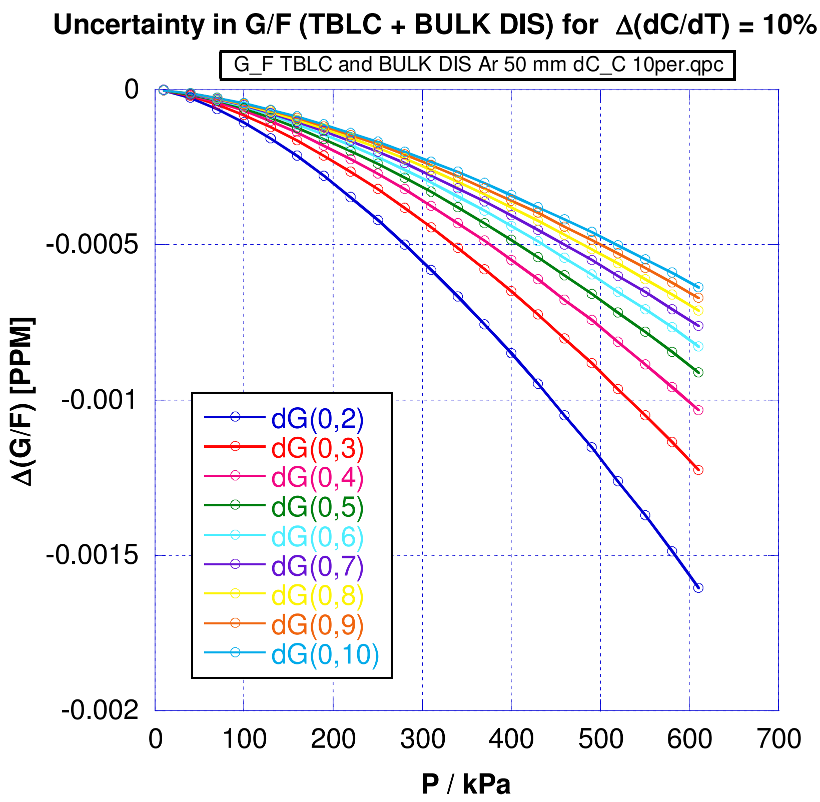

Figure 23

12. U[TBLC/BULK DIS] DUE TO $U(d^2C/dT^2)/d^2C/dT^2 = 10\%$

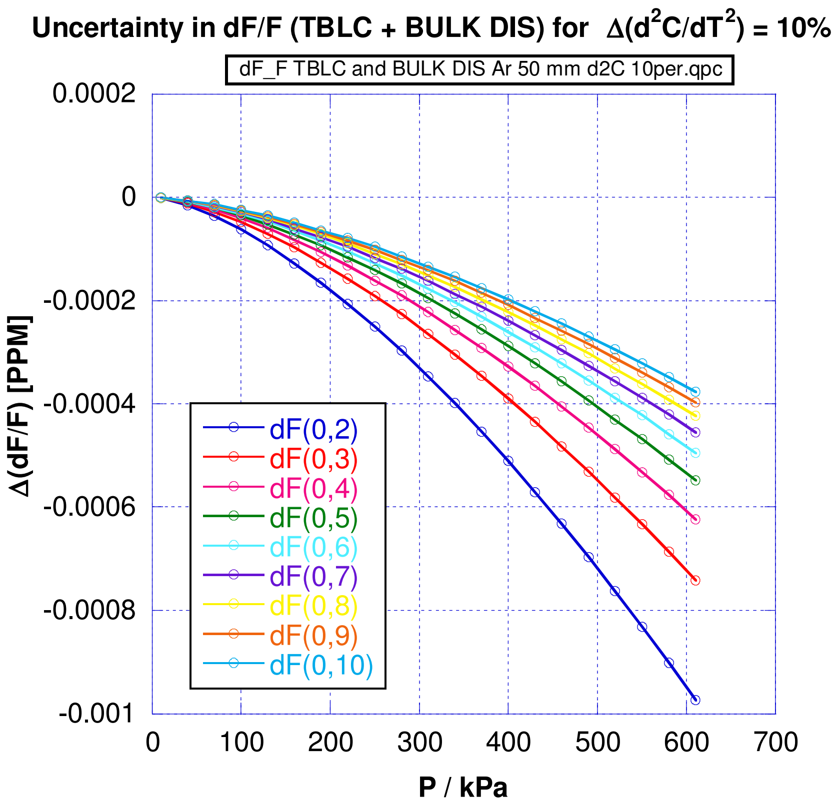

Figure 24

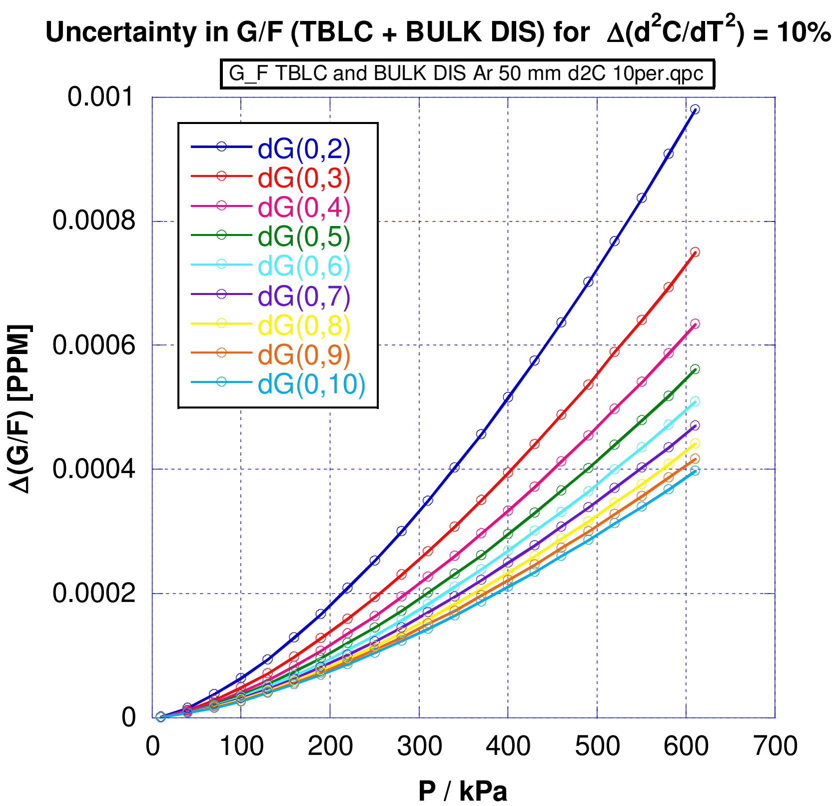

Figure 25

UNCERTAINTY SUMMATION

Assuming all uncertainties are independent, we calculate the combined uncertainty to the TBLC / Bulk dissipation correction by quadrature summation:

Combined fractional uncertainty in frequency correction:

$$\Delta\left(\frac{df}{f}\right)_{\substack{TBLC\\Bulk\ Diss}} = \sqrt{\sum_{i=1}^{n} \Delta_i \left(\frac{df}{f}\right)_{\substack{TBLC\\Bulk\ Diss}}} \tag{1}$$

Combined fractional uncertainty in the halfwidth correction:

$$\Delta\left(\frac{g}{f}\right)_{\substack{TBLC\\Bulk\ Diss}} = \sqrt{\sum_{i=1}^{n} \Delta_i \left(\frac{g}{f}\right)_{\substack{TBLC\\Bulk\ Diss}}} \tag{2}$$

Figures 26 and 27 show the combined fractional uncertainties for the frequency and halfwidth corrections respectively.

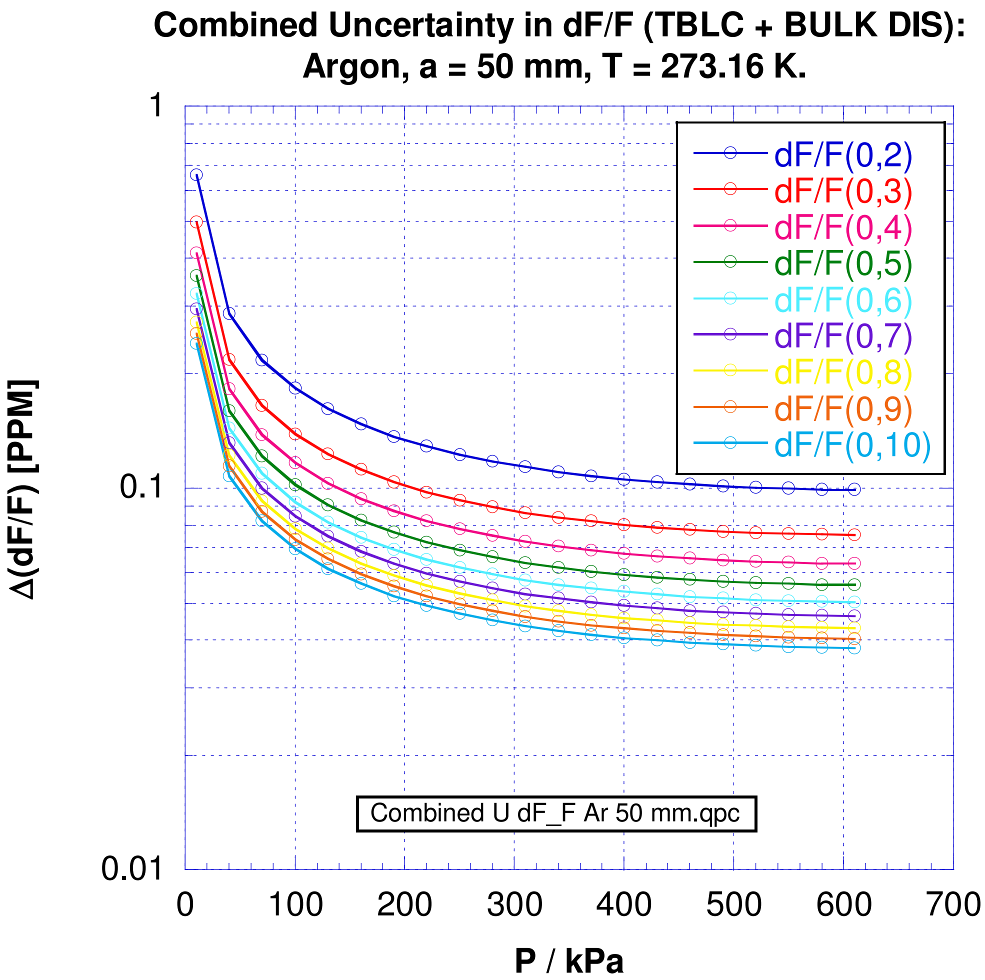

Figure 26

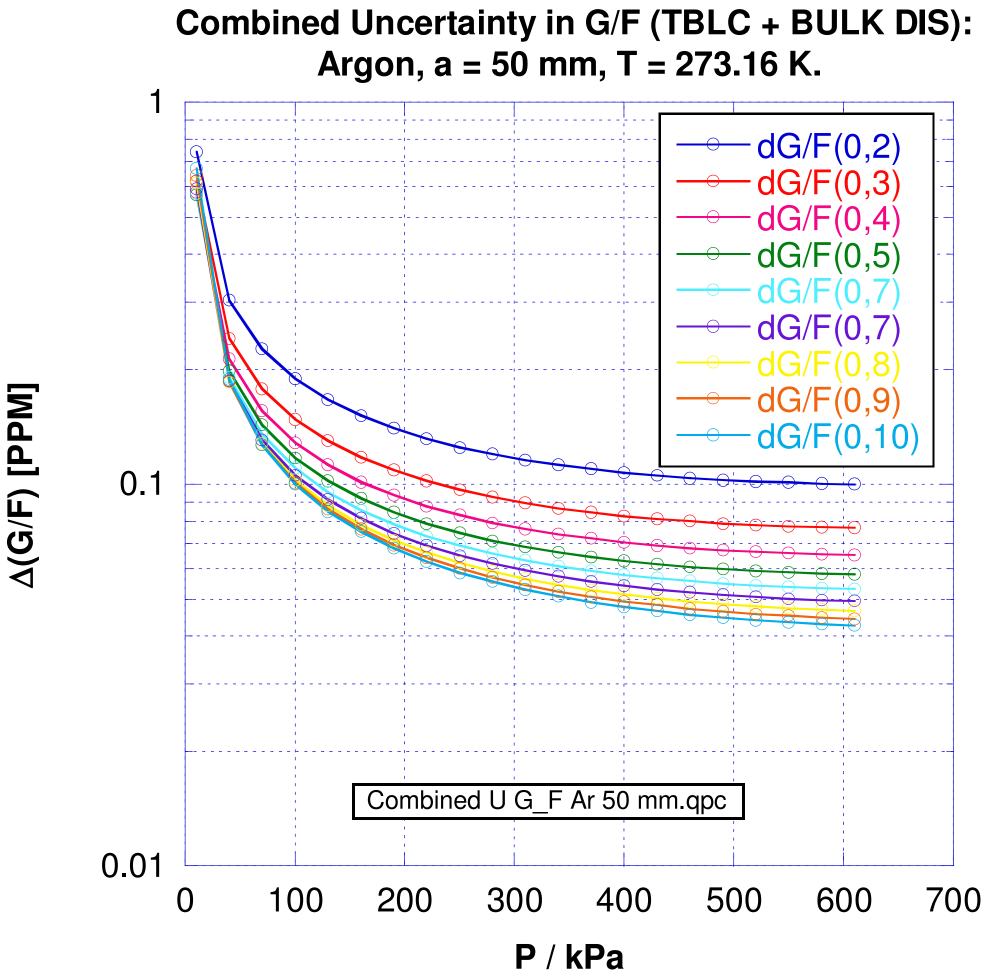

Figure 27

Points to note:

- The largest uncertainties occur at the lowest pressure. This may be expected since the fractional TBLC / Bulk dissipation correction is largest at low pressures.
- The uncertainties are largest for the (0,2) mode reducing for higher order modes. Again, this may be expected since the fractional TBLC / Bulk dissipation correction decreases as the mode number increases.
- At the lowest measurement pressure (50 kPa) the combined fractional uncertainty in TBLC / Bulk dissipation correction varies from:
  - 0.26 PPM to 0.10 PPM for *dF/F* moving from the (0,2) to (0,10) mode.
  - 0.28 PPM to 0.17 PPM for *G/F* moving from the (0,2) to (0,10) mode.
- The reason the uncertainty on *G/F* is slightly larger than that on *dF/F* is due to the fact that the bulk dissipation correction only affects *G/F*.
- At P = 100 kPa, both uncertainties are less than 0.2 PPM.