

Report

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**Uncertainty budget for the
NPL-UCL Swing Arm
Profilometer operating in
comparator mode**

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Uncertainty budget for the NPL-UCL Swing Arm Profilometer operating in comparator mode

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Abstract

This document is a detailed uncertainty budget for the Swing Arm Profilometer, jointly developed by NPL and University College London (UCL) when operating in comparator mode. The various sources of uncertainty are identified and quantified and then summed according to GUM guidelines. The effect on the performance of the instrument is calculated based on a mathematical model. Suggestions are given for ways to improve the instrument performance to reduce the uncertainty to a level where it could be used for E-ELT segment profilometry.

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Approved on behalf of the Managing Director, NPL
by Susan Evans, Director, Industry and Innovation Division

Contents

1	Introduction	1
2	Assumptions	2
2.1	Reference surface assumptions	2
2.2	Assumptions concerning the test surfaces.....	2
2.3	Assumptions on the SAP operation.....	3
3	Basic machine structure	4
4	Contributions to the uncertainty	8
4.1	Reference surface RoC uncertainty.....	8
4.2	Reference surface quality.....	8
4.3	Stability of probe path.....	9
4.3.1	Stability of the PI bearing.....	9
4.3.2	Stability of the main arm system	10
4.3.3	Stability of the X-Y table and vertical column	11
4.3.4	Stability and accuracy of the probe system	13
4.3.5	Stability of probe mount.....	13
4.3.6	Stability of the granite base	13
4.3.7	Basic repeatability and resolution of the SAP measurements	14
4.3.8	Dynamic errors	15
4.4	Repeatability of segment surface location	15
4.4.1	Rotary table stability.....	15
4.4.2	Thermal expansion difference between segments	16
4.4.3	Mounting system repeatability and stability.....	16
4.4.4	Accuracy of segment manufacture	17
5	Summary of uncertainty contributions	18
5.1	Modelling of the probe motion errors	21
5.2	Second order effects.....	24
6	Review of results and suggestions.....	25
6.1	Summary	25
6.2	Future suggestions.....	25
7	Conclusions	28
7.1	Summary of findings.....	28
7.2	Conclusions regarding P-ELT segment metrology	28
8	References	29
	Appendix A – Dimensional drawings of the main SAP components.....	30
	Appendix B – Mathematica® simulation of probing errors	35

1 Introduction

Under the DIUS (formerly DTI) National Measurement System Programme for Length 2002 – 2005 [1], a prototype Swinging Arm Profilometer (SAP) was built as part of NPL's commitment to the SmartOptics Faraday partnership. The SAP was designed and built as a joint project involving NPL and the Optical Science Laboratory at University College London (UCL). The SAP was designed as a generic, multi-range measuring platform for the investigation of new techniques of optical surface metrology. It was used by three PhD students from UCL who were studying designs for hybridised SAP metrology systems for the measurement of large optical aspheric surfaces. The basic SAP instrument was built using components of a Coordinate Measuring Machine that had become surplus to requirements.

Under the follow-on DIUS National Measurement System Programme for Engineering Measurement 2005 – 2008 [2], the SAP was developed further and then delivered to the National Centre for Ultra-Precision Surfaces [3] at OptIC Technium, St Asaph, North Wales, for further evaluation and real-world use. The National Centre is funded by a £4.2 million UK Research Council Basic Technology Project "Ultra-precision surfaces: A new paradigm". This 4 year project aims to advance surface manufacturing, and revitalise UK capability in this, a field of great economic and scientific importance. The project is run by the Optical Science Laboratory at University College London and Cranfield University. The top-level goal is ambitious: to gain a factor of 10 in surface precision and manufacturing time, and address complex aspheric and free-form surfaces.

Since the delivery of the SAP, the Centre has won a contract to manufacture example panels for the forthcoming European-Extremely Large Telescope (E-ELT) [4]. In order to do this, the Centre is having to not only increase the speed of manufacture, but also the speed of measurement. In the current format, the SAP is not suited for measuring such large test optics. The Centre has requested assistance from NPL under the DIUS Metrology For Innovators Programme, to help with the detailed uncertainty budgeting of the existing instrument, with a view to modifying it to enable metrology of E-ELT panels.

This report is the output following a one week secondment of an NPL staff member to OptIC Technium in June 2008.

2 Assumptions

The existing SAP, being designed as a generic, re-configurable instrument, and being based on existing hardware available at the time of build, is not the ideal instrument for measurement of prototype E-ELT (P-ELT) mirror segments. In order to make the machine multi range, the tilt of the precision air bearing had to be adjustable, over a wide range of angles (-30 to +30 degrees). To cope with the air bearing being tilted, various limits were placed on arm length and weight carrying capability. When one operates the SAP, it is clear that the setting precision that is required for P-ELT segments is beyond that available, especially in terms of measuring absolute radius of curvature (RoC).

However, many of these problems can be avoided by operating the SAP in comparator mode against a known concave spherical reference mirror, of the same nominal diameter as the prototype ELT (P-ELT) segment. The modus operandi is to set up the SAP to give a perfect null trace on a previously calibrated master spherical surface, then replace the master surface with the test surface, and repeat the measurements. The SAP then measures the asphericity of the test surface with respect to the master surface. Measurement of the RoC of the master spherical surface is much easier, as it is spherical.

Before continuing with the detailed examination of the sources of uncertainty, it is useful to state the assumptions being made of both the measuring system and the traceability route. It is also important to examine the specification of the P-ELT segments themselves.

2.1 Reference surface assumptions

It is assumed that the spherical reference mirror RoC is:

- known (to a given uncertainty)
- stable
- rotationally symmetric
- very similar to the desired RoC of the segment to be tested

2.2 Assumptions concerning the test surfaces

A typical P-ELT segment is specified as 1234 mm across the flats, which is approximately 1424 mm vertex to vertex (depending on the exact segment geometry) with 50 mm vertex thickness. The maximum vertex to vertex distance across any of the segments to be manufactured is 1450 mm. The typical sag across any segment is about 3.2 mm.

The P-ELT segments will be manufactured from at least 3 materials:

- a. Schott Zerodur, CTE $\alpha = 0.0 \pm 0.2 \times 10^{-7} \text{ K}^{-1}$, density 2.53 g cm^{-3}
- b. LZOS Astrosital, CTE $\alpha = 0.0 \pm 0.6 \times 10^{-7} \text{ K}^{-1}$, density 2.46 g cm^{-3}
- c. Corning ULE, CTE $\alpha = 0.0 \pm 0.15 \times 10^{-7} \text{ K}^{-1}$, density 2.21 g cm^{-3}

The P-ELT segment nominal RoC is $84.000 \text{ m} \pm 0.200 \text{ m}$. All segments are to have a RoC within this tolerance but the maximum segment to segment variation will be considerably less, due to the surface form error specification that is necessary to achieve the wavefront error specification on each segment. The tolerance of $\pm 0.200 \text{ m}$ is effectively a tolerance on the mean RoC of the whole array of segments (to allow for overall focus correction for the telescope as a whole).

The ESO specification for the P-ELT segments [5] gives a maximum allowable wavefront residual error of 30 nm RMS, remaining after removal of:

85% of Z3 (power)

95% of Z4 & Z5 (x & y astigmatism)

85% of Z8 & Z9 (primary spherical, trefoil x)

Separately, there is a specification of 100 nm RMS overall wavefront error before any removal of low order form error occurs. This is the most useful specification to work with for this SAP uncertainty budget, especially as RoC is the same as Z3.

Because this is a mirror surface, the specification on the surface is half that of the wavefront, nominally 50 nm RMS. The conversion from RMS surface to peak to peak depends on the exact nature of the fitted surface and the aberrations. Assuming sinusoidal form, the conversion from RMS to peak to peak is by a $\sqrt{2}$ multiplier, so the 50 nm tolerance becomes 70 nm. Or, on advice from Chris King, this multiplier becomes 3.5 when using Zernike analysis, leading to 175 nm peak to peak surface form error. If it is decided that half of the surface error is allocated to the RoC, this gives a max RoC form error of 35 nm (sinusoid) or 88 nm (Zernike). In order for the uncertainty not to dominate, one should aim for the uncertainty of the measurement of the form error to be at most half of the tolerance, i.e. 17 nm (sinusoid) or 44 nm (Zernike) peak to peak RoC measurement uncertainty.

The ESO specification document [3] states [§4.3.1.6], “*Accuracy of cross-check [technique] shall be such that...undetected error...within the correction range...Table 3.1*”. In other words, this places a slightly looser criterion on the uncertainty of the RoC measurement using the SAP, namely 85% of the RoC surface error, provided that there is an alternative technique that can provide primary RoC measurement. Using 100 nm (RMS) as the 100% value, the ESO specification on the cross-check uncertainty is 85 nm (RMS), if the RoC term is considered in isolation.

2.3 Assumptions on the SAP operation

In its present form, the SAP physically cannot measure P-ELT segments because the arm length is too short for the probe tip to reach the vertex of the segment without the outer edge of the optic at the vertex fouling the X-Y stage during rotation. It is therefore assumed that as a bare minimum change, the arm length will be increased to be sufficient (but also minimised to reduce thermal problems).

Current room temperature control where the SAP is located is approximately ± 2 °C. It is assumed that this will be improved upon by re-building the SAP enclosure and ensuring proper ducting of the motor heat away from the enclosure. The SAP end of the metrology room will be sealed off to slow any rate of change. For the purposes of this budget, a temperature control of the SAP to ± 0.5 °C will be assumed as a typical scenario.

It is assumed that the vibration environment will be essentially quiet during periods of operation of the SAP, since no alternative data exists. If this is not the case, Monte Carlo simulations would be necessary to study the effect of low to medium frequency noise on the analysis. The only adjunct to this is the occasional resonance exhibited by the SAP arm during some scans, which will need averaging of data to remove.

3 Basic machine structure

In the following description, reference is made to the SAP design drawings by Simon Oldfield, e.g. [SAP15]. These are attached in reduced form in Appendix A.

The SAP is currently housed in a part of the metrology room at the National Centre for Ultra-Precision Surfaces, OpTIC Technium, St Asaph, Wales. It is as-delivered, following design and manufacture at NPL and delivery to Wales. As befits technology developed under NMS Programmes, the SAP was constructed as a general purpose research instrument and it is now to be modified specifically for use on P-ELT metrology. This uncertainty budget will use the existing structure for the uncertainty budget, but make one or two assumptions about necessary modifications to the machine, such as extending the effective arm length, which are absolutely necessary. In some cases, additional, desirable, modifications will be suggested.

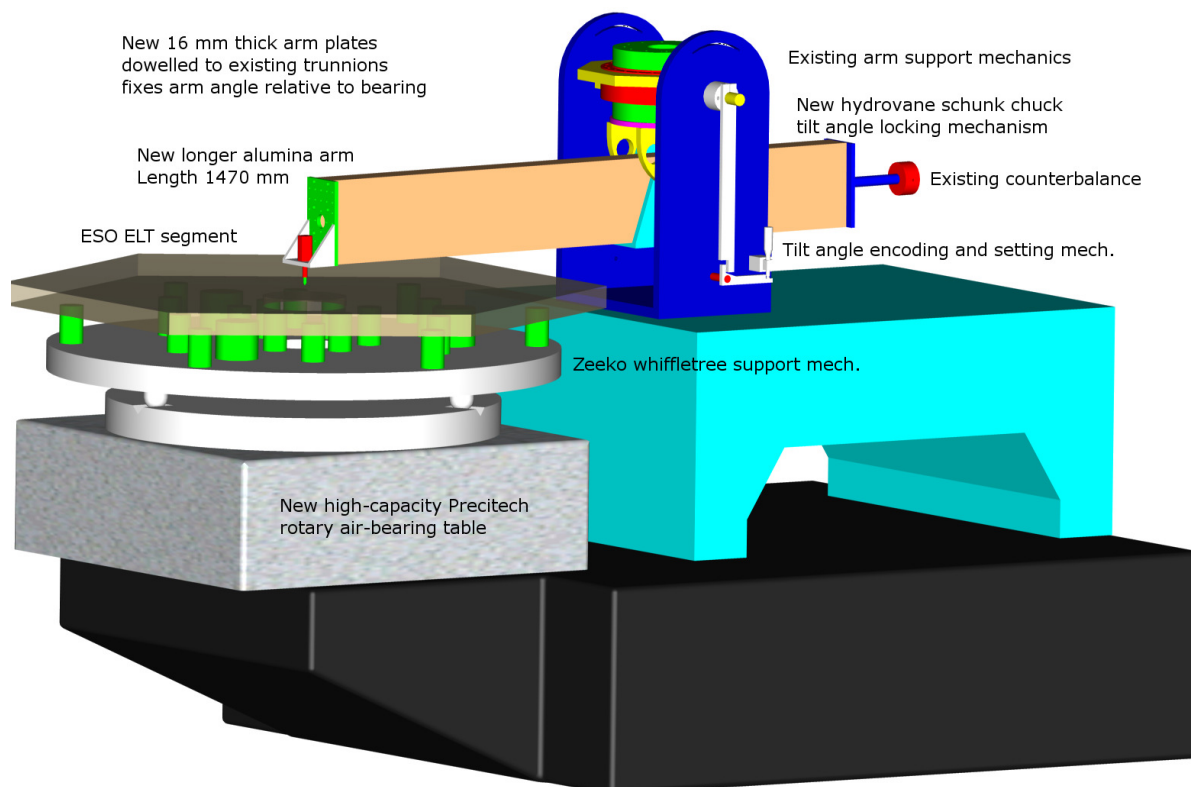


Figure 1 - general assembly of the modified SAP

The SAP consists of a large granite base upon which sit the Horstmann rotary table (which will need to be replaced by a new table, probably from Precitech) and the X-Y table. On top of the X-Y table is the arm support system and the arm itself. The segment currently sits directly on the top of the Horstmann bearing but when the bearing is replaced, a whiffle tree support will be used instead. The proposed (modified) SAP is shown in figure 1.

The metrology loop, which runs from the probe tip, through the optic, through the mechanical supports and bearings and back to the probe via the arm, therefore comprises:

- the P-ELT segment
- whiffle tree
- Horstmann (or replacement) bearing
- granite base
- moving stage and column
 - X-Y stage
 - side cheek plates [SAP16] and brace plates [SAP21, SAP23]
 - stub axles [SAP10]
- PI bearing
- main arm system
 - the arm centre plate [SAP13]
 - trunnion plates [SAP12]
 - arm plates [SAP11]
 - alumina arm and mounting plate
 - probe mounting bracket
 - the probe itself

It is important to note that parts of the structure are suspended whereas others are supported from below. Thermal expansion of materials will therefore self-cancel to a certain extent in the vertical direction.

The following are the general assembly drawings of the SAP, for reference when considering the metrology loop. Details of individual components can be found in Appendix A.

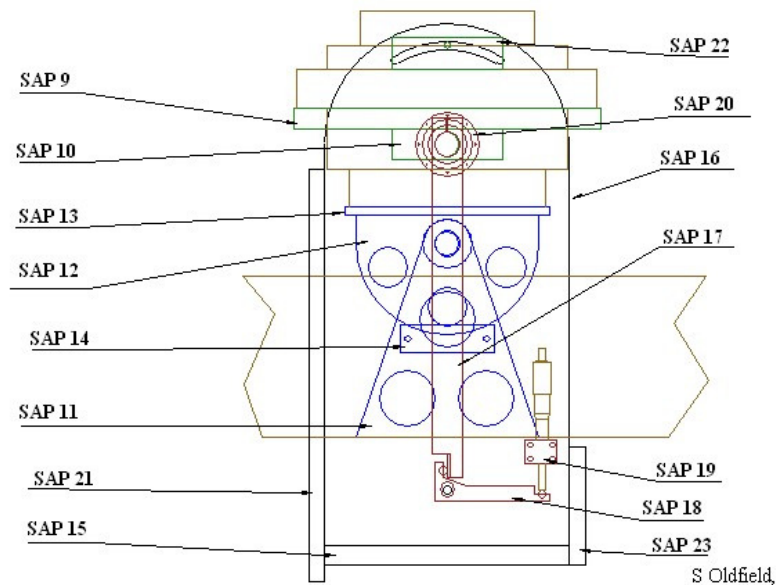


Figure 2 - SAP arm support - side view

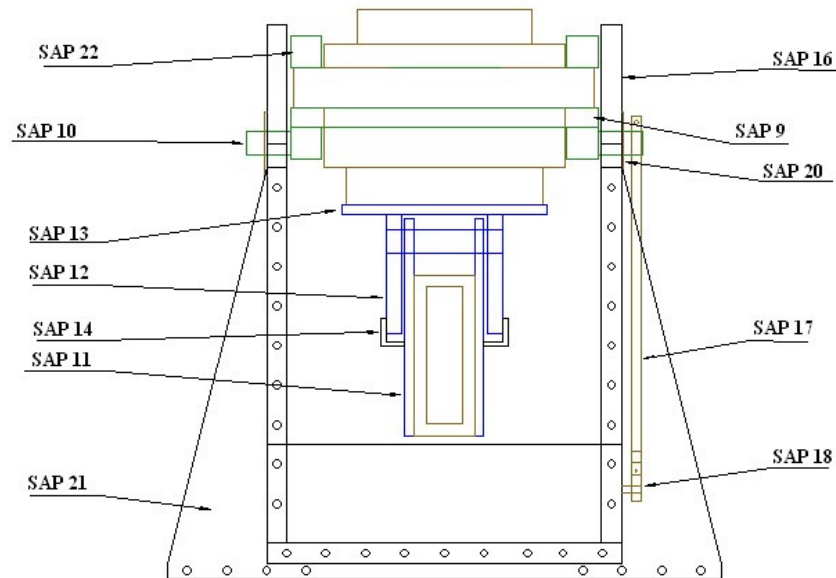


Figure 3 - SAP arm support front view

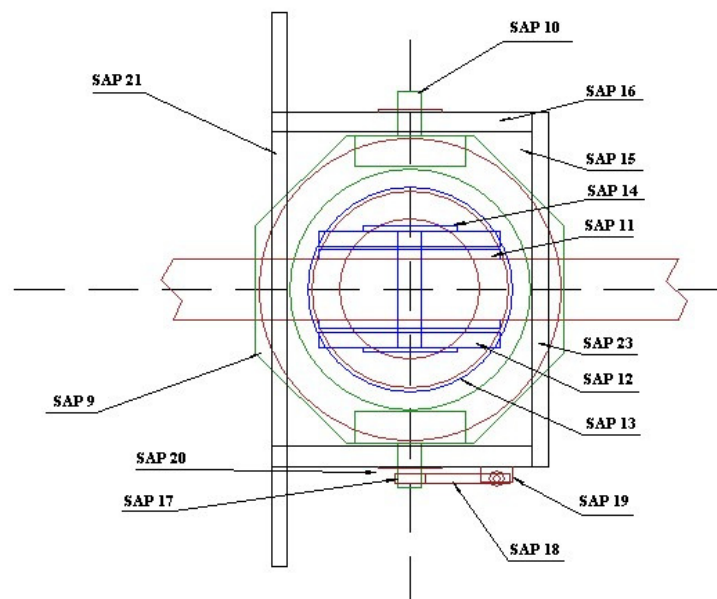


Figure 4 - SAP arm support top view

The SAP operates by moving the tip of a probe across the surface to be measured, and thus measures the difference in height of the surface with respect to a reference surface generated by the rotation axes of the SAP. Provided that the SAP tilt (θ) remains stable, the SAP arm length remains fixed and the rotary bearing used for the scan suffers no parasitic tilt, then the reference surface upon which the probe tip moves is that of a perfect sphere. The actual probe path is an arc on the surface of the reference sphere, running through the vertex of the segment.

In comparator mode, if the SAP is set up to give a null trace on the reference sphere, with the probe normal to the surface of the reference surface, then a measurement of a test segment

will be a measurement of the departure of the test segment's surface from that of the reference sphere, i.e. the asphericity of the test surface.

The basic operational equation of the SAP is

$$\sin \theta = L'/R$$

where θ is the tilt angle of the SAP bearing, L' is the effective arm length and R is the nominal RoC of the test or reference segment. Note that L' is measured normal to the rotation axis of the precision air bearing, and is only exactly equal to the physical arm length, L , when the bearing is being used with its axis vertical and the arm horizontal, i.e. with the arm normal to the bearing axis.

The basic sources of uncertainty for the SAP operating in comparator mode for RoC determination are:

- uncertainty of reference segment RoC
- quality of the reference segment surface
- stability of the probe path between reference and test optic measurements
- repeatability of locating the segment surfaces
- noise and uncertainty of the probe

In the analysis below, uncertainty components appear in several formats, depending on their nature. For example, some items are random in nature and these are specified in the format ' $\pm xx$ ' to indicate that they are considered for quadratic summation.

Some contributions (mostly thermal expansion) have a specific direction and because some of these may cancel, they are given according to a sign convention, as follows. A positive thermal contribution moves the probe upwards with respect to the granite surface, or moves the segment surface downwards, i.e. increases the probe-surface gap. A positive horizontal expansion component moves the probe and segment closer together along the granite table.

4 Contributions to the uncertainty

4.1 Reference surface RoC uncertainty

It is assumed that the uncertainty of the RoC of the reference segment is not a direct contribution to the uncertainty on the measurement of the difference in RoC between this and the test segments, using the SAP. In other words, the absolute value of the RoC of the test segments is not of interest (provided that they are all sufficiently close to each other and provided that they are all close to that of the test segment, and all are well within the 200 mm tolerance on the nominal value).

Nil contribution (in comparator mode for comparing across the set)

4.2 Reference surface quality

The reference surface needs to be manufactured to a sufficiently high quality that it can have an accurate RoC determination made and that the error in shape shall not unduly influence the settability of the SAP on this reference surface. In other words, the first stage of measuring a P-ELT RoC is setting the SAP to give a null trace on the reference sphere. If the reference surface is truly spherical, the trace will be absolutely flat and this is a very good fiducial setting. If the reference surface has any significant asphericity over the line of the SAP trace, this will influence the setting of the SAP tilt and arm length, leading to an error of the reference base radius setting. Whilst the absolute value of this error will not lead to an error in the comparator results (provided it is well away from the 200 mm specification on the nominal RoC), variability of the setting will contribute to the uncertainty, as each time the SAP is set on the reference segment, it will be set to a different base radius. It is highly likely that the SAP will need re-setting on the reference segment from time to time (probably between each P-ELT measurement) so this is a contributing uncertainty.

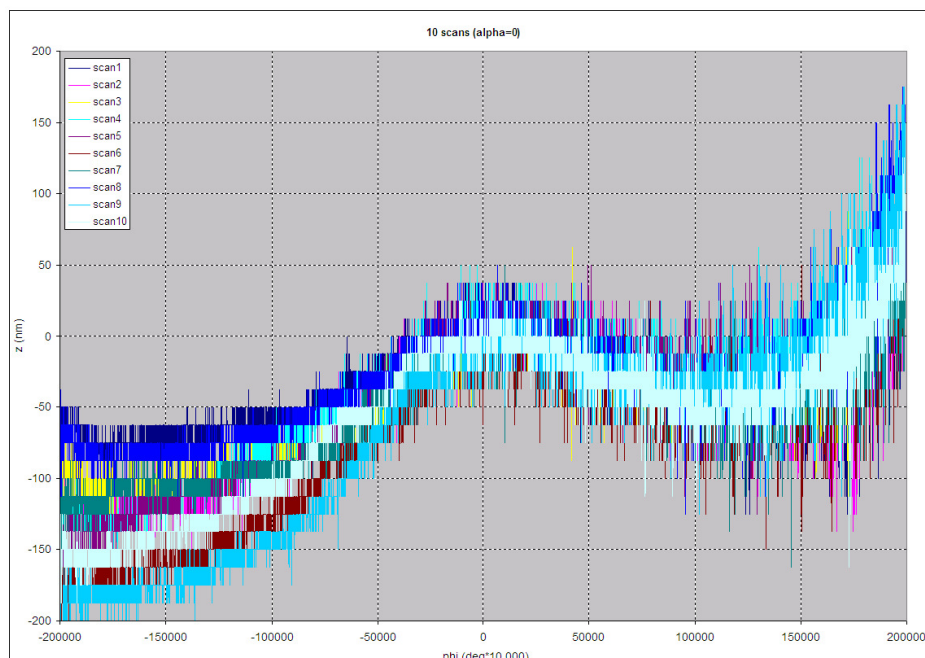


Figure 5 - repeated scans at NPL

The actual uncertainty contribution is hard to enumerate as it will depend on the exact form error of the master component (asphericity). Experience with the SAP on a good quality spherical surface at NPL has shown that the SAP can be set to repeat the same path to give traces which agree to within approximately 8 probe counts (12.5 nm each) or 100 nm. The

variability in the scans was mostly due to change in tilt of the Horstmann table with respect to the swing arm scan axis. After correcting for this, a repeatability of setting on the correct combination of arm length and tilt angle seems to be around 2 to 3 probe counts, i.e. 25 to 38 nm. If this was a trace obtained on the reference segment, it should be possible to reset the SAP alignment to give traces which look the same to this order of accuracy.

Error in master profile: ± 38 nm form error on the surface

4.3 Stability of probe path

This is the most critical and most difficult uncertainty to control due to the large size of the SAP. Variation of the probe tip motion from the correct nominal path leads directly to errors in the measured surface form. Depending on the direction of the probe 'wander' this may lead to an error in the perceived RoC. The probe path considered in this section is with respect to the granite base of the SAP – any issues concerned with the stability of the segment surface with respect to the granite base are handled separately, in a later section.

The stability of the probe path depends on several items, each of which is examined in detail below.

4.3.1 Stability of the PI bearing

The specifications of the PI bearing, which is probably the highest quality commercial air bearing available, are as follows:

Axial runout:	< 25 nm total
Radial runout	< 25 nm total
Tilt:	< 0.1 μ rad ($1 \times 10^{-7} L$ error)
Stiffness:	350 N/ μ m radial
	1750 N/ μ m axial
	11.3 Nm/ μ rad tilt

As the PI bearing is only being rotating about one quarter of a rotation, it is unlikely it will experience the full range of the radial runout. At constant pressure and loading, it is also unlikely to see much axial runout as axial motion will be negligible. These numbers are supported by the data shown by testing the SAP at NPL. The runout and tilt errors are likely to be very repeatable. There is unlikely to be any change in the loading of the arm between measurements on the reference and test segments, and so the probe path will be repeatable, as far as the bearing errors are concerned. The only uncertainty contributions therefore will be in the form of distortion of the reference segment profile due to bearing errors causing parasitic probe motion during the setting on the master segment. Provided these parasitic errors are repeatable, the effect will simply be that of a mis-setting of the tilt angle/arm length combination of the SAP, leading to a global error in the segments' RoC.

Considering that the PI bearing will be operating with the axis almost vertical, the radial runout will have the same effect as a change in the arm length and the axial runout will appear as a vertical probe error. The tilt error will appear as either a tilt of the trace, or as a parasitic tilt of the SAP tilt axis, θ . As the SAP uses only one quarter of the rotation range, the worst case scenario is that we would experience 12.5 nm radial and axial runout errors and 0.025 μ rad tilt error.

The radial runout will appear as a change in the arm length, and since the SAP is set up with the arm almost horizontal, this will be very close to 12.5 nm arm length change. The axial runout will appear as a vertical movement of the probe and the 0.025 μ rad tilt error will also

appear as a vertical movement of the probe tip, by an amount equal to the effective arm length $\times \sin 0.025 \mu\text{rad}$, or approximately 23 nm.

Error in master profile: $\pm 36 \text{ nm}$ vertical & $\pm 12.5 \text{ nm}$ arm length change

4.3.2 Stability of the main arm system

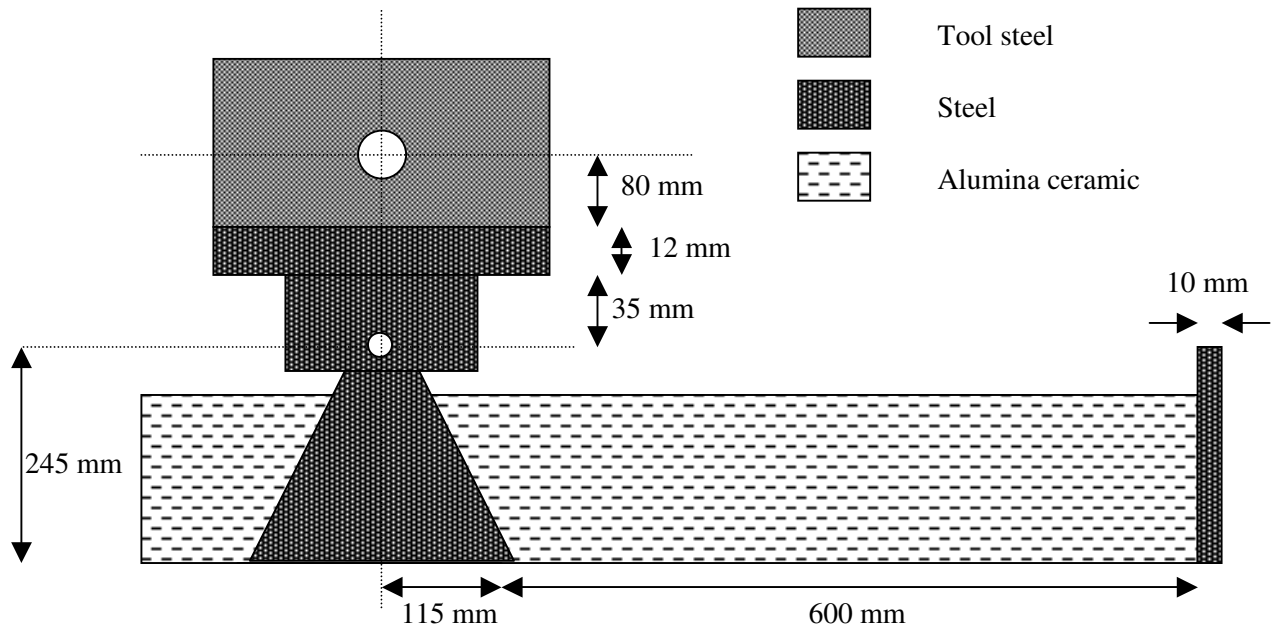


Figure 6 - The main arm system

Figure 6 shows the dimensions of the metrology loop of the existing main arm system, with the bearing axis set vertical ($\theta = 0^\circ$). (This ignores any part of the metrology loop associated with the probe and its mount, which are dealt with separately). When the bearing axis is tilted, there is a $\cos \theta$ effect on the upper 3 dimensions. The arm support plates are glued onto the sides of the alumina beam. The assumption is that the vertical structure of the whole of the arm support plates expands, rather than taking the mid-line through the centre of the beam. Using a minus sign as vertical expansion is positive in the upwards direction and these items are suspended and therefore expand downwards, gives:

Horizontal: $+715 \text{ mm alumina} + 10 \text{ mm steel}$
 Vertical: $-245 \text{ mm steel} - (127 \text{ mm steel}) \times \cos \theta$

The arm, as it stands, is not long enough for the probe to reach the vertex of the P-ELT. The distance from the rotation axis to the outer face of the probe mounting plate is 725 mm when the rotation axis is vertical, however 90 mm of this is lost due to the protruding front section of the X-Y table, and another 165 mm is lost due to the offset of the bearing axis from the front of the cheek plates (see below) leaving an effective clear arm span of 470 mm (the distance clear of the bearing axle). For a 1450 mm maximum diameter hexagon, a clear arm span of at least 725 mm is necessary.

If we choose an absolute minimum clear arm span of 725 mm, then add on the required 90 mm + 165 mm protrusion clearance, this gives a physical arm length of 980 mm from outer edge of probe plate to a point midway along the arm clamping plate.

When the PI bearing is tipped forwards for measurement of concave optics, and the arm is re-levelled, the probe tip effectively moves away from the vertex, back towards the X-Y table by

an amount given by $127 \text{ mm} \times \sin \theta$. This means that the arm physically needs to be longer to reach the vertex. So the physical arm length as defined above needs to be a minimum of $980 \text{ mm} + (127 \times \sin \theta) \text{ mm}$.

Now, using the basic SAP equation of $\sin \theta = L'/R$, where L' is the effective arm length (approx 980 mm) and R is the segment RoC (84 m), then θ is 0.67 degrees. Then the term $(127 \times \sin \theta)$ is 1.5 mm, which is a small correction to the minimal arm length, taking the absolute minimum length to 981.5 mm. Comparing this with the existing length of 725 mm shows that a minimum extension of approximately 255 mm is required. In practice larger clearance is desirable, but any probe mount is likely to be of sufficient size to cover this additional spacing requirement.

With a minimal arm length, $L_{\min}$, of 981.5 mm (from rotary axis to end of mounting plate) and with a tilt angle of 0.67 degrees, the minimum effective arm length, $L_{\min}'$, is then given by

$$L_{\min}' = (L_{\min} + 245 \tan \theta) \cos \theta = 984.3 \text{ mm}$$

(note that this is an approximation since this new value for L' would require an adjustment to θ and therefore an iterative solution process – as the arm gets longer, the tilt angle needs to increase and the X-Y table needs to back away from the segment).

Now the above vertical and horizontal distances of the metrology loop can be revised to be:

Horizontal: $+715 \text{ mm} + 256.5 \text{ mm (extension) alumina} + 10 \text{ mm steel}$
 Vertical: $-245 \text{ mm steel} - (127 \text{ mm steel}) \times \cos \theta$

These dimensions are referenced from the PI bearing tilt axis.

If it is assumed that all the components respond to temperature changes in a linear fashion, and at the same rates, with thermal expansion coefficients of $11.7 \times 10^{-6} \text{ K}^{-1}$ for the steel, $6.1 \times 10^{-6} \text{ K}^{-1}$ for the alumina, then the change to the metrology loop is given by:

Horizontal:	$+5.93 \mu\text{m}/^\circ\text{C} + 0.117 \mu\text{m}/^\circ\text{C} \text{ C} = +6.05 \mu\text{m}/^\circ\text{C}$
Vertical:	$-4.35 \mu\text{m}/^\circ\text{C}$

4.3.3 Stability of the X-Y table and vertical column

This part of the structure is actually more complicated than shown as there is an intermediate body that sits on air pads to allow motion control using crossed air bearing slideways. However, the mechanical structure is lifted on four air bearing pads at the corners of the X-Y table and it is this that is part of the metrology loop. However there is a slight complication in that the X-Y table will expand with temperature, forcing the feet to spread further apart, and the top column is supported on three feet bolted to the X-Y table, and the separation of these feet will change with temperature as well.

The feet of the X-Y table are free to slide along the granite but the intermediate body is fixed against a slideway bolted to the granite table. As the temperature changes, the X-Y table will push against the intermediate body which will push against the slideway. The exact change to the metrology path will depend on the position of the intermediate body with respect to the slideway, and this depends on the position of the X-Y table on the granite base.

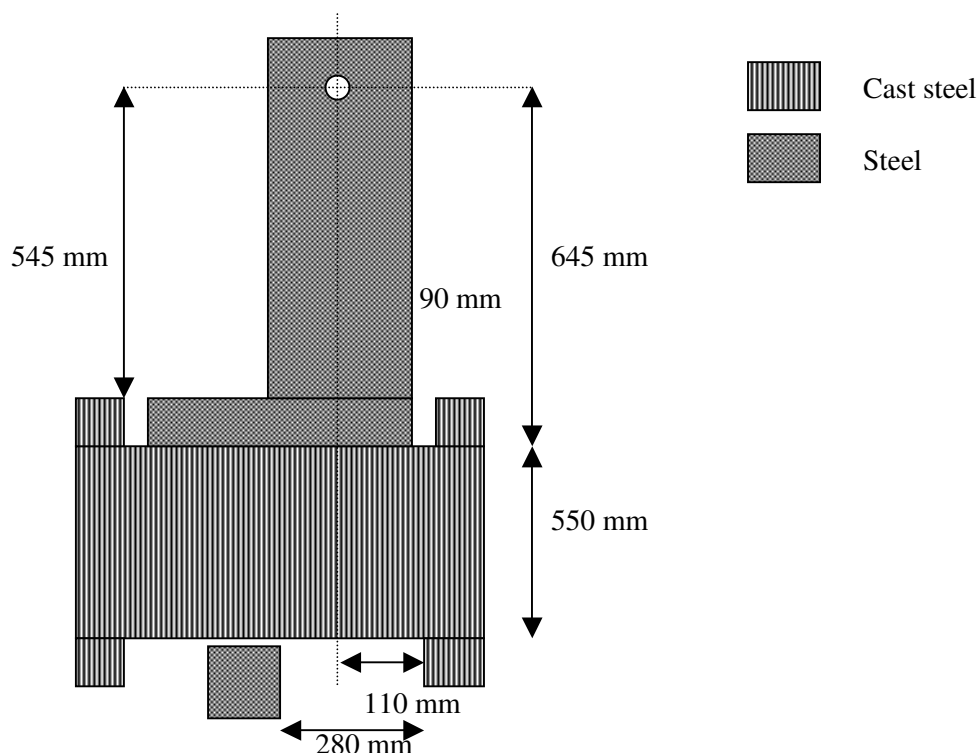


Figure 7 - the X-Y table and vertical column

Setting the X-Y table at the likely location for P-ELT segment measurement allows some approximate dimensions to be determined. The front pair of the feet on the top column is just above the front air bearing pads of the intermediate body and this location is 280 mm ahead of the slideway fixed to the granite base. The tilt axis of the PI bearing is 110 mm behind the front air pad. This means that there is $280 - 110 = 170$ mm of steel between the fixed slideway and the PI rotation axis, in the horizontal direction. All the structures are made of steel and to first order they can be assumed to expand equally about their centres, at equal rates and the feet of the vertical column will maintain contact with the same locations on the X-Y table (they are in fact bolted). Therefore, with reference to the granite base, the PI bearing axis will move forwards (towards the optic) as the temperature increases due to an effective 170 mm steel path length.

Previous tests have shown that the X-Y table is not stable when supported on the air pads as any variation in air pressure leads to a change in ride height of the pads. It is recommended that after initial alignment of the X-Y table, it is left with the air off and allowed to settle. This will require any further adjustments to the SAP to be made elsewhere, e.g. via a small stage on the end of the arm.

Therefore in the vertical direction, between the granite base and the PI tilt axis, there is 500 mm of cast steel and 645 mm of tooling steel. This entire structure will expand upwards as the temperature rises.

Horizontal: +170 mm cast steel

Vertical: +500 mm cast steel, + 645 mm tooling steel

If we assume that all the components respond to temperature changes in a linear fashion, and at the same rates, with thermal expansion coefficients of $11.7 \times 10^{-6} \text{ K}^{-1}$ for tooling steel, $15 \times 10^{-6} \text{ K}^{-1}$ for the cast steel, then the change to the metrology loop is given by:

Horizontal:	+2.63 $\mu\text{m}/^\circ\text{C}$
Vertical:	+7.50 $\mu\text{m}/^\circ\text{C}$ +7.55 $\mu\text{m}/^\circ\text{C}$ = +15.05 $\mu\text{m}/^\circ\text{C}$

4.3.4 Stability and accuracy of the probe system

At the moment the probe system is a Solartron LT12. The LT12 probe head contains a high-accuracy optical scale, together with a mechanical bearing assembly. For maximum accuracy, a fused quartz glass substrate is used to minimise temperature effects and a reference mark is included on the scale to facilitate absolute measurement. The existing model has a 12.5 nm interpolated scale pitch (10 μm actual pitch). The specified accuracy of the probe is 500 nm with 100 nm repeatability. Calibration of the probe at NPL against a laser interferometer showed classical interpolations errors (0.06 μm peak to valley sinusoid on a 5 μm pitch) and a linear scaling error (0.2 μm in 12 mm). Assuming a maximum asphericity of 1.51 mm [6] for any P-ELT, this is considerably larger than the pitch of the interpolation error and so the full 0.06 μm peak to valley error will be visible in the profiles. This is not removable since although the exact phase dependence with respect to the scale lines is fixed, the thermal expansion of the scale and the probe holder will change the offset with respect to the zero position of the reference surface.

Since the probe will be required to travel over a range of 1.5 mm, the measured linearity of 0.2 μm in 12 mm will also contribute an amount of 20 nm error, which appears as a vertical displacement.

The thermal expansion of the unit is quoted as $-0.4 \mu\text{m K}^{-1}$. Since the probe is almost perfectly vertical for the whole of its travel, this expansion will appear as a vertical displacement.

The calibration also showed a tendency of the probe stem to flip-flop against the bearing edges, requiring that all traces are made uni-directional with a pre-travel across the surface to guarantee correct alignment of the linear scale against the bearing. It would be useful therefore to replace the probe with one of less travel, better mechanical stability and finer resolution about the zero point, e.g. a low force LVDT, ideally with long retract capability.

Vertical:	$\pm 60 \text{ nm}, \pm 20 \text{ nm},$
Vertical:	$-0.4 \mu\text{m}/^\circ\text{C}$

4.3.5 Stability of probe mount

At the moment the probe mount is a rather large aluminium structure, necessary to cope with different surface heights and geometries encountered by the SAP. For P-ELT metrology, the design will be a fixed design and its contribution to the uncertainty budget will have to be determined. With careful consideration, it may be possible to use the probe mount to cancel some of the expansion of the vertical column structure (see later).

4.3.6 Stability of the granite base

A possible longer time contribution to the uncertainty would be due to expansion of the approximately 1180 mm gap between the centre of the rotary table and the fixed slideway underneath the intermediate body. Taking a nominal CTE for granite to be $5 \times 10^{-6} \text{ K}^{-1}$, this would lead to a horizontal change to the metrology loop of around $+5.9 \mu\text{m}/^\circ\text{C}$.

However there is need to consider the specific heat capacity of granite compared with those of the other materials in the SAP, and to take into account the mass of the granite block (ignoring emissivity values).

Material	Specific heat capacity
Aluminium alloy	$0.9 \text{ Jg}^{-1} \text{ K}^{-1}$
Steel	$0.5 \text{ Jg}^{-1} \text{ K}^{-1}$
Alumina	$0.8 \text{ Jg}^{-1} \text{ K}^{-1}$
Granite	$0.9 \text{ Jg}^{-1} \text{ K}^{-1}$

The granite block size is approximately $2.9 \times 1.9 \times 0.5 \text{ m}$, and with a density of 2.75 g/cm^3 , this block has a mass of approximately 7.6 tons. The heat capacity of this block is therefore 6.84 MJ K^{-1} . Compare this with the alumina arm which is 40 kg at $0.9 \text{ Jg}^{-1} \text{ K}^{-1}$, or 3.6 kJ K^{-1} . For the same thermal step in the environment, the granite will change temperature at a rate 2000 times slower than the arm. Assuming that the temperature in the room is cyclical of low amplitude, the rate of change of temperature of the granite will be effectively zero.

As far as operating in comparator mode is concerned, the contribution to the uncertainty from the granite base is considered negligible due to the thermal time constant of the granite preventing fast changes to geometry from thermal effects.

Nil contribution on short to medium time scales in a cyclical temperature environment

4.3.7 Basic repeatability and resolution of the SAP measurements

The data shown in Figure 5 indicates that over short time scales, the SAP repeatability is currently about 8 probe counts (12.5 nm each) or 100 nm. The variability in the scans was mostly due to change in tilt which was suspected to come from the poor quality of the Horstmann rotary table rather than movement of the vertical column. After correcting for this, a repeatability of around 2 to 3 probe counts, i.e. 25 to 38 nm was achieved which is well within the combined uncertainties of the various components.

One item to be included which has so far been omitted is the basic resolution of the various metrology axes which are operational during repeated scans.

It is assumed that the tilt axis, θ , will be clamped rigidly using the Schunk chucks and since this will then not be adjusted (after setting on the reference segment) the resolution of the tilt adjustment micrometer and bell crank play no further part. When clamped, the tilt angle is controlled by the stability of the Schunk chuck. As these devices operate radially symmetrically and because they are mounted centred on the PI tilt axis, it is anticipated that they will not impart any rotation to the tilt axis as the temperature changes. If it is found that the helical structure of the chuck clamp causes rotation under thermal change, it may be possible to add a second pair of clamps with opposing helices to counteract this effect. As the SAP will be used in comparator mode, with the tilt remaining unchanged between scans, the resolution in the tilt scale adjustment also has no effect.

It is probably a good idea to back off the tilt adjust micrometer when the Schunk chucks have been locked, so that the only structures that can influence the tilt are the chucks.

The sweep angle is measured using the rotary scale mounted on the PI bearing. This scale is interpolated to 0.36 arc second resolution. It is not anticipated that the resolution of the scale will make a significant contribution to the uncertainty for RoC measurements because RoC is a larger scale form error. The same arguments hold for the errors in this scale.

The rotary table currently has an angular reading interpolated to 1.8 arc seconds. During a scan, the table is not rotated but the table needs to be set to the correct rotation before the scan to ensure that the trace runs across the correct part of the surface. Mis-rotation of the table will appear as a form error in the expected profile. The maximum astigmatic term for a P-ELT is 1.5 mm. So, in a quarter rotation of the optic (90°) there will be a $\sim 1.5 \text{ mm}$ change in the

probe reading. The ‘asphericity gradient’ is thus $16.7 \mu\text{m}/^\circ$ rotation, so an angular setting error of 1.8 arc seconds leads to an 8 nm error in the probe reading at the worst part of the segment.

Vertical:	$\pm 8 \text{ nm}$
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4.3.8 Dynamic errors

The dynamic errors of the SAP such as twisting of the arm beam due to the load of the end probe mount are quite small because the tilt angle, θ , is small so the arm path is nearly horizontal, and the arm will be rotationally balanced with counterweights. In any case, the dynamic errors are likely to be very repeatable so they will cancel when the SAP is used in comparator mode provided no alterations are made to the loads or the geometry.

The only remaining dynamic error is likely to be noise due to vibrations. The SAP has previously shown a resonance in the last 30 degrees of several scans that leads to high frequency errors in the measured profiles. This is random in nature and can be reduced by repeated measurements.

Nil, plus some random noise (approx 50 nm peak to valley) to be averaged out
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4.4 Repeatability of segment surface location

There are four contributors to the segment surface location repeatability: the stability of the Horstmann/Precitech rotary table, the thermal expansion difference of the test segment with respect to the reference segment, the repeatability of the mounting system; the differential offsets of the two segments’ surfaces with respect to their mounting systems.

4.4.1 Rotary table stability

At the moment the limiting factor is the poor stability of the rotary table. It is planned to replace this with a better design. Testing at NPL revealed the current table to have a vertical stiffness of $100 \text{ N}/\mu\text{m}$, tilt stiffness of $10 \text{ Nm}/\mu\text{rad}$. The vertical stiffness is ten times less than that of the PI bearing. The table has also been shown to exhibit wobble and poor repeatability and will need to be replaced. If we assume the replacement has a similar specification to the PI bearing, then the effective terms that will contribute to the uncertainty will be the tilt sensitivity and the axial and radial runouts. Because of the slightly different masses and mass distributions of the different P-ELT segments, each one will have to be balanced on the rotary table to achieve the horizontal level of the table surface. For now, the PI bearing specifications of $1750 \text{ N}/\mu\text{m}$ axial stiffness and $11.3 \text{ Nm}/\mu\text{rad}$ tilt sensitivity will be used. For the variation in mass, a maximum 1.5 mm asphericity on a 1.45 m diameter part is negligible but the density variations are more significant:

Density of Zerodur	2.53 g cm^{-3}
Density of Astrosital	2.46 g cm^{-3}
Density of Corning ULE	2.21 g cm^{-3}

This leads to a 14% variation in mass between the Zerodur and ULE items. A 14% variation, in a mass which is approximately 208 kg, if uncorrected would lead to a difference in table ride height of 120 nm. It is assumed that the table can be loaded with extra masses to both balance the mass load symmetrically and achieve similar masses across the different materials. Since masses are available in various sizes and the mass distributions and values of the items can either be calculated or measured, it is assumed that this uncertainty can be reduced to a negligible level.

The only remaining items would then be the runouts. Careful design of the mounting whiffle trees could ensure that the rotary table does not need rotating far for aligning the segment with the SAP (i.e. choosing to set the segment's symmetry line such that it is coplanar with the SAP bearing axis (i.e. symmetrical trace)). The table could also be rotated to the same angles with the reference surface on top, to check for runout errors on the reference traces.

The only contribution then from the rotary table would be the instability due to air supply pressure changes and the mass balancing of the load. This is likely to be similar to the short term scan repeatability seen on the SAP at NPL, namely of the order of 25 nm.

In any case, whichever rotary table is used it has physical height and is typically made of cast steel, which will be subject to thermal expansion. The maximum height of the rotary table (and any underside support) is limited by the need to fit the whiffle tree and segment on top with a slight clearance from the underside of the SAP arm beam. In fact 'shimming' the rotary table to the maximum height has a distinct advantage that it can cancel the thermal expansion of the equivalent height of the X-Y table and vertical column. Since all of these are made (or could be) of steel-like materials, it should be possible to cancel all but the thickness of the optic itself. One would need to be careful to allow for the opposite expansion of the SAP arm and PI bearing (which expand downwards).

For now, the dimension of the Horstmann table will be used, namely a thickness of 421 mm, subject to an expansion coefficient of $11.7 \times 10^{-6} \text{ K}^{-1}$.

Vertical:	$\pm 25 \text{ nm}, -4.9 \mu\text{m}/^\circ\text{C}$
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4.4.2 Thermal expansion difference between segments

Although the test and reference segments have the same nominal thermal expansion coefficient (namely zero), there may be slight variations between them due to different manufacturing batches, and certainly between the different materials, e.g. Zerodur vs Astrosital. The specifications on the materials are given in the first page of this document. The worst case differential expansion is $\alpha = 0.0 \pm 0.6 \times 10^{-7} \text{ K}^{-1}$ for Astrosital. The vertex thickness above the whiffle tree pads is 50 mm, so this leads to a differential vertical expansion of 3 nm $^\circ\text{C}$. Differential radial expansion will be larger at 87 nm across the diameter, however on local scales this is negligible when performing fitting to find the RoC.

Vertical:	$\pm 3 \text{ nm}/^\circ\text{C}$
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4.4.3 Mounting system repeatability and stability

Assuming the whiffle tree is made of steel-like materials and is approximately 200 mm in height, this adds a thermal instability of $-2.34 \mu\text{m}/^\circ\text{C}$ (negative because it brings the surface closer to the probe). Additionally, if typical kinematic locating devices such as balls in vees are used, then one can assume a vertical and radial repeatability of the order of a few micrometres, say 5 micrometres. If a rotationally symmetric kinematic mount is used, then the centering should be very good, perhaps to a micrometre or so. Remaining errors would therefore be vertical.

In terms of the Z positioning, it will be necessary to adjust the height of the segment to match the height of the test segment on the rotary table, otherwise the height difference will contribute to an effective change in arm tilt and arm length. This requires either an absolute probe reading from the SAP probe or the SAP probe reading to be maintained 'live' during swapping of the master and test parts. Provided this is the case, the vertical repeatability of

the kinematic mount is not an issue, and the resetability is given by the resolution of the probe, i.e. 12.5 nm.

Horizontal:	$\pm 1 \mu\text{m}$
Vertical:	$\pm 12.5 \text{ nm}, -2.34 \mu\text{m}/^\circ\text{C}$

4.4.4 Accuracy of segment manufacture

The main item to be considered here is the centering error of the reference feature used to align the P-ELT to the rotation axis of the SAP. It was previously proposed that the back face central cavity could be used for this purpose, as this will also be used to align the part when being polished (the cavity will be pre-machined before delivery of the pre-formed blanks). The specification on this cavity is that it should be concentric to the vertex normal to within 200 μm , ideally 100 μm . However each segment will be individually measured by Cranfield using their large Leitz CMM. Taking the MPE of $(1.9 + L/400) \mu\text{m}$, for measurement of a 1.45 m part on a Leitz PMM 30.20.10, the uncertainty over the full segment diameter is expected to be 5.5 μm . Taking half of this (the cavity is in the centre, so the maximum edge to cavity distance is half the segment diameter), gives 2.8 μm .

This centering error could contribute to either the X or Y axis positioning error of the part on the rotary table. Depending on the rotation required for alignment with the SAP, this could appear in either axis, so on average a component of $1/\sqrt{2}$ will appear in each axis.

Variations in height (thickness) of segments can be taken into account by the height adjustment described above.

Horizontal:	$\pm 2.0 \mu\text{m}$
Horizontal (orthogonal):	$\pm 2.0 \mu\text{m}$

5 Summary of uncertainty contributions

Reference surface RoC uncertainty

Nil contribution (in comparator mode for comparing across the set)

Reference surface quality

Error in master profile: ± 38 nm form error on the surface

Stability of the PI bearing

Error in master profile: ± 36 nm vertical & 12.5 nm arm length change. The change in arm length leads to a negligible uncertainty.

Stability of the main arm system

Horizontal instability: $+6.05 \mu\text{m}/^\circ\text{C}$

Vertical instability: $-4.35 \mu\text{m}/^\circ\text{C}$

Stability of the X-Y table and vertical column

Horizontal instability: $+2.63 \mu\text{m}/^\circ\text{C}$

Vertical instability: $+15.05 \mu\text{m}/^\circ\text{C}$

Stability and accuracy of the probe system

Vertical: ± 60 nm, ± 20 nm, $-0.4 \mu\text{m}/^\circ\text{C}$

Stability of the probe mount

To be calculated after the new mount is designed. Careful design could be used to compensate for excessive thermal expansion errors elsewhere.

Stability of the granite base

Nil contribution on short to medium time scales in a cyclical temperature environment that is controlled at the 0.5°C level.

Basic repeatability and resolution of the SAP measurements

Vertical: ± 8 nm

Dynamic errors

Nil contribution in comparator mode, plus some random noise to be averaged out

Rotary table stability

Vertical: ± 25 nm, $-4.9 \mu\text{m}/^\circ\text{C}$

Thermal expansion difference between segments

Vertical: $\pm 3 \text{ nm}/^\circ\text{C}$ i.e. negligible

Mounting system repeatability and stability

Horizontal: $\pm 1 \mu\text{m}$

Vertical: ± 12.5 nm, $-2.34 \mu\text{m}/^\circ\text{C}$

Accuracy of segment manufacture

Horizontal: $\pm 2.0 \mu\text{m}$

Horizontal (orthogonal): $\pm 2.0 \mu\text{m}$

Source	Magnitude	Assumed distribution, divisor to 1 std deviation	Horizontal magnitude at 1 std deviation	Vertical magnitude at 1 std deviation
Reference surface quality	±38 nm form error on master	Rectangular, $\sqrt{3}$	-	22 nm form
PI bearing stability	±36 nm form error on master ±12.5 nm arm length change	Rectangular, $\sqrt{3}$	±8 nm arm length change	21 nm form
Main arm stability	H: +6.05 $\mu\text{m}/^\circ\text{C}$ V: -4.35 $\mu\text{m}/^\circ\text{C}$	Rectangular ¹ , $\sqrt{3}$ Rectangular, $\sqrt{3}$	+3.49 $\mu\text{m}/^\circ\text{C}$ -	- -2.51 $\mu\text{m}/^\circ\text{C}$
Table & column stability	H: +2.63 $\mu\text{m}/^\circ\text{C}$ V: +15.05 $\mu\text{m}/^\circ\text{C}$	Rectangular, $\sqrt{3}$ Rectangular, $\sqrt{3}$	+1.52 $\mu\text{m}/^\circ\text{C}$	- +8.69 $\mu\text{m}/^\circ\text{C}$
Probe system stability ²	V: ±60 nm, ±20 nm, -0.4 $\mu\text{m}/^\circ\text{C}$	Rectangular, $\sqrt{3}$; Normal, 1; Normal, 2	-	±41 nm, -0.2 $\mu\text{m}/^\circ\text{C}$
Angle table resolution	V: ±8 nm	Rectangular, $\sqrt{12}$	-	±5 nm
Rotary table stability	V: ±25 nm, -4.9 $\mu\text{m}/^\circ\text{C}$	Normal, 1; Rectangular, $\sqrt{3}$	-	±25 nm, -2.9 $\mu\text{m}/^\circ\text{C}$
Mounting stability	H: ±1 μm V: ±12.5 nm, -2.34 $\mu\text{m}/^\circ\text{C}$	Normal, 1 Rectangular, $\sqrt{12}$; Rectangular, $\sqrt{3}$	±1 μm	±3.6 nm, -1.35 $\mu\text{m}/^\circ\text{C}$
Segment manufacture	H: ±2.0 μm (X and Y)	Normal, 2	±1.0 μm (X and Y)	-

Table 1 - summary of uncertainty contributions

¹ In Table 1, the thermal contributions have been converted into changes in probe or surface locations based on the assumption of a temperature instability which will be specified as a rectangularly bound distribution

² In Table 1, the specification of the probe thermal expansion coefficient was not given with any distribution or coverage factor. Normal distribution and 2-sigma coverage are therefore assumed.

The remaining calculations are at the one sigma level, unless otherwise stated.

The first stage of combining the uncertainties is to take into account the positive and negative thermal expansion contributions. Fortunately, many of the vertical contributions cancel to give the following overall uncertainty contribution.

Horizontal (X): $+5.01 \mu\text{m}/^{\circ}\text{C}$
 Vertical (Z): $+1.73 \mu\text{m}/^{\circ}\text{C}$

Next, the contributions to the master surface profile errors, are summed quadratically to give $\pm 31 \text{ nm}$ form error and $\pm 8 \text{ nm}$ arm length error.

Next there are the non-temperature dependent errors in the horizontal and vertical directions which are again summed quadratically to give:

Horizontal (X): $\pm 1.4 \mu\text{m}$
 Horizontal (Y): $\pm 1.0 \mu\text{m}$
 Vertical (Z): $\pm 48 \text{ nm}$

Now, if it is assumed that the temperature stability in the laboratory is $\pm 0.5 ^{\circ}\text{C}$ (rectangular bounds, as stated above) then the temperature dependent uncertainties become:

Horizontal (X): $+2.51 \mu\text{m}$
 Vertical (Z): $+0.87 \mu\text{m}$

Adding these in quadrature with the temperature independent uncertainties, gives the following:

Horizontal (X): $\pm 2.9 \mu\text{m} \pm 8 \text{ nm arm length change}$
 Horizontal (Y): $\pm 1.0 \mu\text{m}$
 Vertical (Z): $\pm 0.87 \mu\text{m} \pm 31 \text{ nm master surface form error}$

As far as the probe to surface distance is concerned, a change in arm length of 8 nm is almost exactly the same as relative motion of the probe tip with respect to the surface, in the horizontal direction. So these two elements can be combined, though the overall change is negligible. (In fact, using the simulation, the effect is shown to be sub nanometre probe error due to the change in arm length).

TOTALS

Horizontal (X): $\pm 2.9 \mu\text{m}$
 Horizontal (Y): $\pm 1.0 \mu\text{m}$
 Vertical (Z): $\pm 0.87 \mu\text{m}, \pm 31 \text{ nm master surface form error}$

5.1 Modelling of the probe motion errors

A simplified model of the SAP was constructed in Mathematica[®] to study the effect of the parasitic error motion on the probe readings. The source code of the model can be found in Appendix B. The model uses the following input parameters and is based on measurement of a spherical surface:

$$R = 84.0 \text{ metres}$$

$$L = 0.9843 \text{ metres}$$

$$\theta = \sin^{-1}(L/R)$$

The model computes the distance from the segment centre of curvature to the probe tip and compares this with the distance from the centre of curvature to the segment surface, which should be equal to the base radius of curvature, R . The model assumes that the surface of the segment is spherical and that the parasitic error motions occur at the SAP bearing rather than at the segment, i.e. relative motion.

Using the model with a $2.9 \mu\text{m}$ X error gives:

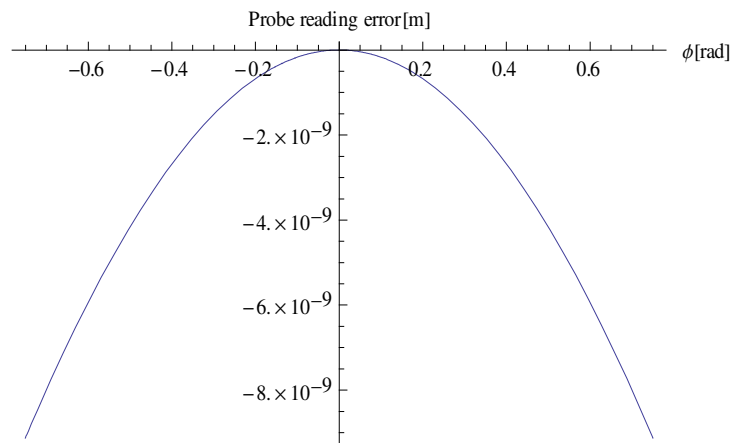


Figure 8 - probe reading error due to $2.9 \mu\text{m}$ X shift

And with a $1.0 \mu\text{m}$ Y error gives:

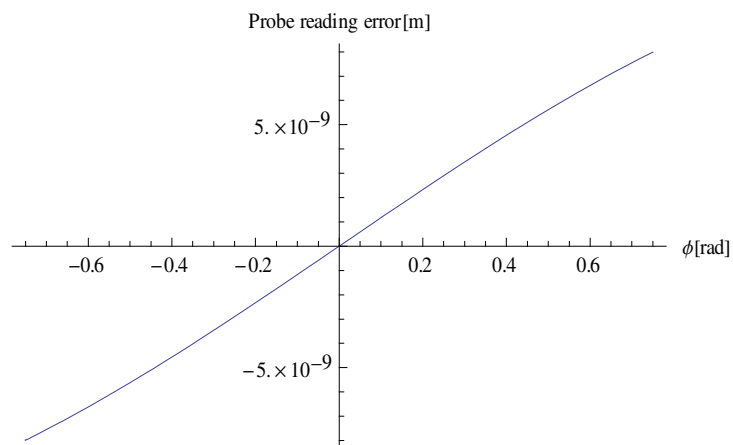


Figure 9 -probe reading error due to $1.0 \mu\text{m}$ X shift

Combining the two gives:

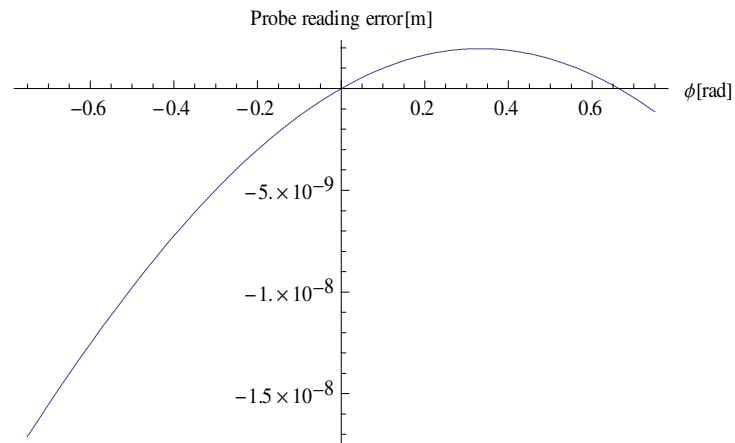


Figure 10 - probe reading error due to 2.9 μm X and 1.0 μm shifts

Next, giving a direct 0.87 μm Z error gives:

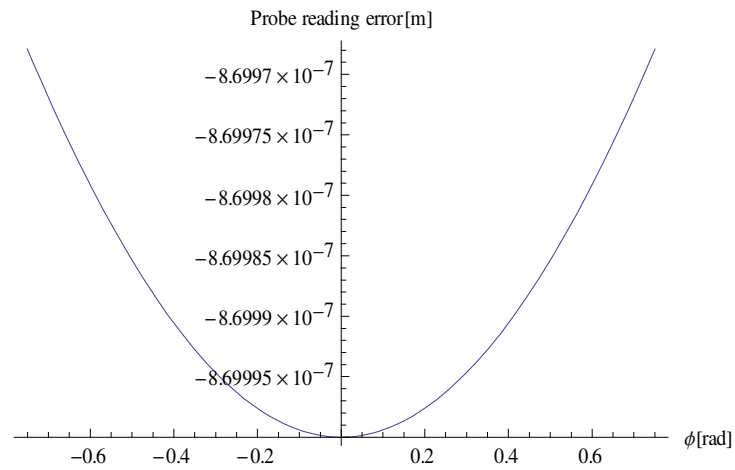


Figure 11 - probe reading error due to 0.87 μm Z shift

All three errors (X, Y, Z) combined:

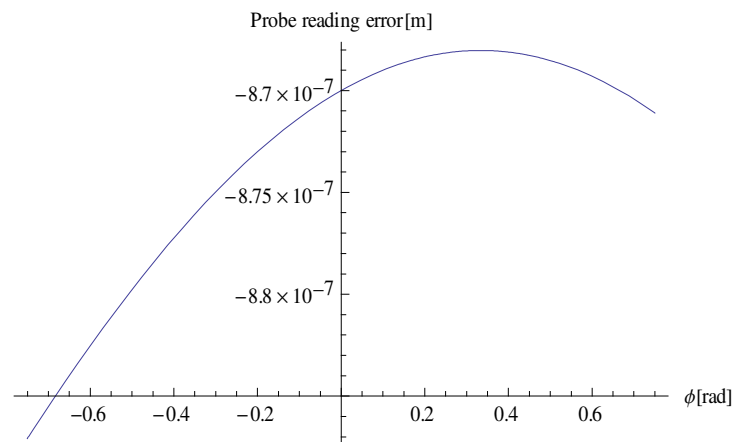


Figure 12 - probe reading error due to X, Y, Z shifts

In a 3D view, it is clear that the probe error based on the combinations of X, Y and Z error motions has both tilt and curvature components:

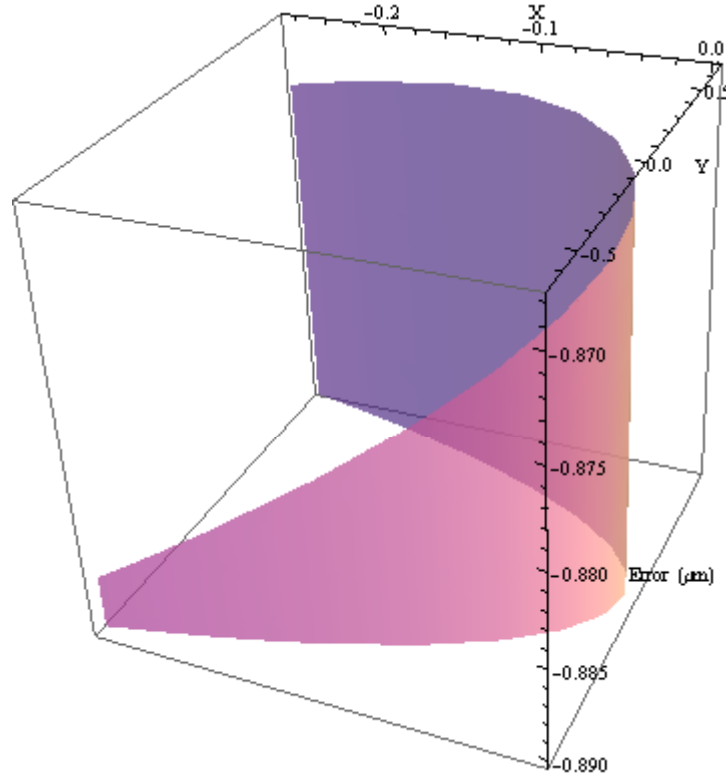


Figure 13 - 3D plot of probe error from combined X, Y, Z shifts

Note that in figure 11, the Z error has led to an offset on all probe readings equal to the Z error, plus a very small (sub nanometre) additional curvature due to the change in the effective arm length. The auto scaling of the plot exaggerates the curvature. In practice, provided the Z error is slow to occur (long time scale) then the setting of the segment height to null the probe reading will eliminate the probe offset reading, though the small additional curvature will remain (see below).

If one were to encounter the trace shown in figure 12, when expecting a flat, null trace, the conclusion would be that the segment was not level and had a RoC error, when in fact the errors were due to the SAP itself. Looking at figures 8 through 12, the worst RoC error appears to come from the erroneous 9 nm additional sag at the centre of the segment profile in figure 8 (the errors caused by Y and Z shifts either lead to tilt or smaller RoC errors). For a spherical mirror, the central sag, s , and the cross-flats diameter, d , are related to the RoC, R , by:

$$R^2 = \left(\frac{d}{2}\right)^2 + (R - s)^2 \text{ or } R = \frac{d^2}{8s} + \frac{s}{2}$$

For a spherical mirror of $R = 84.000$ m, and $d = 1450$ mm, the solution to these equations gives $s = 0.003\,128\,778\,5$ m. Changing s by 9 nm leads to a solution for R of $R = 83.999\,973$ m, i.e. an error of $26.8\,\mu\text{m}$ in the RoC. This can also be obtained from the sensitivity of R to changes in s , given by

$$\frac{dR}{ds} = \frac{1}{2} - \frac{0.262813}{s^2}$$

which for $s = 0.003$, gives a value of 26847. Clearly, the RoC determination is very sensitive to errors in s . This is a commonly encountered problem – the determination of radii from partial arcs is prone to large errors, even when small errors are encountered in determination of the arc's shape.

This calculation above holds for spherical segments. For aspherical segments, there is an additional contribution from the offsets multiplied by the asphericity gradients. As stated previously, the maximum astigmatic term for a P-ELT is 1.5 mm. The pathological case where this causes the biggest error is when the arc swept out by the probe is aligned with its major axis parallel with one of the astigmatism axes. Then at the extreme ends of the swing, the offset of the probe will be in the direction of highest gradient. Assuming half of 1.5 mm is the astigmatism over one radius of the segment, then the asphericity gradient is of the order of $1.5 \text{ mm}/725 \text{ mm}$, or $2.07 \mu\text{m}/\text{mm}$. For a probe displacement of $2.9 \mu\text{m}$, the asphericity gradient displaces the probe vertically by 6 nm. So, to an order of magnitude, the additional errors due to the asphericity are similar to those from considering a spherical surface being mis-probed (9 nm form error on spherical surfaces plus an additional 6 nm of asphericity gradient error).

The errors dealt with in most detail are those concerning the X and Y locations of the probe with respect to the segment surface. In order to calculate the effect of the vertical displacement, one or two assumptions are necessary. Firstly the majority of the vertical uncertainty is related to unbalanced thermal expansion between the surface support and the SAP main structure. This is expected to be a slowly varying component due to the thermal masses involved. However, it will take some time before any of the reference or master segments is stable on the SAP after swapping them, during which time the temperature of the structure may change.

In theory, any vertical change in relative height between the probe and the surface would be visible as an offset to the probe readout when sitting on the apex of the test surface compared to the zero reading on the reference surface. The alignment procedure assumes that the test segment is height adjusted during setup (ideally as soon as possible after being swapped for the reference segment). Any significant thermal drift thereafter would appear as a change to the probe readout. This could be cancelled using the height adjustment, prior to scanning the test segment.

If the height is not adjusted and the probe reading changes, this will have two effects. Firstly, the scan traces will all be shifted vertically by an offset equal to the thermal drift (but this will not change the profile shape). Secondly, it changes the effective arm length by an amount equal to the vertical displacement $\times \sin \theta$. For a vertical change of 870 nm, the effective arm length changes by 10 nm. An arm length change of 10 nm, leads to a profile error of less than a nanometre.

5.2 Second order effects

The main analysis has considered essentially perfect knowledge of the various contributions, e.g. a perfect knowledge of the thermal expansion coefficients and a perfect match between the expansions of the various steel components. In practice this is unlikely and so the second order terms, i.e. the uncertainty in the expansion coefficient (or the difference across the various steel components) $\times$ the temperature change, ought to be considered. However without detailed knowledge of these materials, it is not possible to make a reliable estimate of their uncertainty contributions. As an order of magnitude estimate, one could allow an additional 50% of the first order summed contribution (to be added in quadrature).

6 Review of results and suggestions

6.1 Summary

Detailed examination of the various uncertainty components has revealed that the instability of the swing arm probe with respect to the segment surface leads to probe position errors on the segment surface of 1.0 μm and 2.9 μm in the Y and X directions, respectively. There is also a vertical error of 0.87 μm but this may be removed during setting on the test sphere. There is also likely to be an error in the form of the master surface of the order of 31 nm, which will affect the ability to set the correct null of the probe on this master component.

These values are given at one sigma, i.e. a confidence level of approximately 68% and assume input parameters have been determined with effectively infinite degrees of freedom.

Taking into account the radius of curvature of the segments that are to be measured, these probe instabilities lead to form errors of magnitude 19 nm (on spherical surfaces) which appear as a mixture of parasitic tilt and curvature error, depending on the exact analysis used.

However, the 19 nm error is based on calculations on a spherical surface. The actual segments are strongly aspheric and an order of magnitude calculation shows that there is **likely to be a doubling of this error due to the asphericity gradients encountered by the probe**. There will be additional uncertainty terms due to second order effects, but these have not yet been determined. **Thus one should anticipate a 38 nm minimum uncertainty on form measurement of P-ELTS (one sigma level), perhaps rising to around 44 nm when second order terms are included.**

It is typical to use a 95% confidence interval, which requires a coverage factor, k , of 1.96 (for effectively infinite degrees of freedom assumed in this analysis). So the 95% confidence level uncertainty for the SAP measurements on aspheric segments is around 87 nm.

The SAP therefore is not able to achieve the required form accuracy specification as set out in §3.2.1.2 and §3.2.1.3.(5) of the ESO specification [5], as it is likely to be double the 44 nm uncertainty tolerance discussed in section 2.2 of this report. In other words, the SAP uncertainty is similar to the form tolerance, hence it is unable to prove or disprove compliance with the tolerance.

The SAP is, however, on the borderline of being able to fulfil the requirements of §4.3.1.6, i.e. the physical cross-check, which needs an instrument with less than 85 nm form error.

In terms of radius of curvature measurement, it must be born in mind that the sensitivity to segment sag errors is the limiting factor when using a partial arc determination of the radius of curvature. The tolerance on absolute radius of curvature across the segments, of 200 mm translates to a 67 μm absolute form accuracy (for the sag). The SAP would be able to achieve this, however it is clear that the form accuracy tolerance is more critical and not yet achievable. Aside from the form accuracy tolerances given above, there is no explicit tolerance on radius of curvature matching between the segments.

6.2 Future suggestions

The uncertainty budget has already assumed that the Horstmann rotary table is replaced by something of similar specification to the PI bearing. Therefore, in order to make further improvements in the stability of the SAP, it would be necessary to:

- thermally insulate the main arm (as this is the main component of the X error);
- match the heights of the steel parts of the SAP main body and the rotary table, to cancel the majority of the Z error (though this is a smaller contribution);
- improve the fiducialization of the segment with respect to the rotary table.

Improved temperature control beyond the suggested 0.5 °C limit would also be beneficial, but its effectiveness will ultimately be limited by the probe resolution and the temperature-independent uncertainties. The above calculations and simulations showed that the likely form error on a spherical surface would be of the order of half the allowed tolerance, and there would likely be additional errors due to the asphericity gradients on strongly aspheric surfaces, such as P-ELT panels, leading to an overall uncertainty approximately equal to the form tolerance., perhaps slightly more once second order terms are included. Metrology of P-ELTs would require the uncertainty reducing by a factor or two to three, in order for the results to be useful.

This could be accomplished by a number of initiatives:

- Improved thermal shielding of the instrument, particularly the steel X-Y table and the ceramic arm;
- Temperature monitoring of these structures at various positions to assess the areas which need greatest shielding;
- Improved control to the room as a whole;
- Matching of the height of the thermal path on the segment support to that on the SAP X-Y and arm structure;
- Fitting of a probe mounting stage, with inverted thermal path (backwards facing).

However it should also be noted that the value of 19 nm form error is less than twice the resolution of the current probe (12.5 nm resolution). If the SAP is modified to reduce the overall uncertainty due to the machine stability, then the probe resolution of 12.5 nm will become the limiting factor. To progress beyond a 6.25 nm uncertainty (half the resolution) would require fitting a probe with better resolution. For P-ELTs, the maximum asphericity is just over 1.5 mm so a probe of 2 to 3 mm range and nanometric resolution would be needed.

Recapitulating the summation of the uncertainties, there are both temperature-independent terms:

Horizontal (X):	$\pm 1.4 \mu\text{m}$
Horizontal (Y):	$\pm 1.0 \mu\text{m}$
Vertical (Z):	$\pm 48 \text{ nm}$

and temperature-dependent terms:

Horizontal (X):	$+5.01 \mu\text{m}/^\circ\text{C}$
Vertical (Z):	$+1.73 \mu\text{m}/^\circ\text{C}$

With an assumed $\pm 0.5 \text{ }^\circ\text{C}$ temperature variation, the temperature-dependent terms become:

Horizontal (X):	$+2.51 \mu\text{m}$
Vertical (Z):	$+0.87 \mu\text{m}$

Of these, it is the Horizontal (X) term which leads to the largest form errors. The Horizontal (Y) error appears as mostly parasitic tilt. Vertical (Z) error has a smaller effect.

If the temperature-dependent terms are reduced to zero, there are still the temperature-independent terms, of which the 1.4 μm Horizontal (X) error is the worst, the source of which is the mounting stability. If the temperature deviations were reduced from $\pm 0.5\text{ }^{\circ}\text{C}$ to $\pm 0.28\text{ }^{\circ}\text{C}$ then the temperature-dependent terms would reduce to

Horizontal (X):	+1.4 μm
Vertical (Z):	+0.5 μm

i.e. the temperature-independent term in X would start to dominate. In practice, temperature control of laboratory-sized volumes to better than $\pm 0.2\text{ }^{\circ}\text{C}$ level is quite difficult, so $\pm 0.2\text{ }^{\circ}\text{C}$ would seem a difficult but achievable target for the overall volume.

Stability of the arm structure at the $0.1\text{ }^{\circ}\text{C}$ level could then be achieved with improved thermal insulation. In which case the X error would reduce to the root sum square of $\pm 1.4\text{ }\mu\text{m}$ and $\pm 0.5\text{ }\mu\text{m}$, i.e. $\pm 1.49\text{ }\mu\text{m}$. With this value of Horizontal (X) error, the simulation predicts 5 nm form error. Doubling this to allow for asphericity gradient leads to 10 nm (or 20 nm at 95% confidence level, $k = 1.96$). In section 2.2 it was stated that the target for the uncertainty of the measurement should be at most half of the tolerance, i.e. 17 nm (sinusoid) or 44 nm (Zernike) peak to peak RoC measurement uncertainty.

So with the temperature control improved and the arm insulated to provide $0.1\text{ }^{\circ}\text{C}$ temperature stability, and the probe replaced, the uncertainty of the form measurements could be reduced to less than half the tolerance, i.e. useable.

In the longer term, it would be beneficial to consider a more bespoke solution to the problem of measurement of a large number of segments with large radii of curvature. The present SAP was designed as a multi-range instrument and compromises were made to its design to enable this. It was also designed as a form measuring instrument with the intent of using it to carry various types of probes and to be adjustable.

The E-ELT segments, although of many different specific designs, are all quite similar when it comes to overall diameter and base radius of curvature. It should be possible to design a bespoke measuring system which is not adjustable, and is therefore easier to set up and more stable than the existing SAP. A detailed study should be performed to assess alternative measuring systems which could be used for such segments. For example, NPL has previously built profilometers based on angle-scanning and integration, for the measurement of nearly- flat surfaces [7]. It has also used the process of multiple-beam angle scanning and data redundancy for the *in situ* mapping of flatness and orthogonality errors of mirror systems [8, 9, 10, 11]. NPL has also performed work on the calibration of wavefront curvature sensors [12], which could be mounted on a modified swing arm system.

7 Conclusions

7.1 Summary of findings

This report has examined the uncertainty contributions expected to be encountered when operating the NPL-UCL Swing Arm Profilometer in comparator mode. Some recommendations have been made regarding ways to improve the performance of the SAP to enable it to perform useful metrology of prototype ELT segments.

The uncertainties manifest themselves as parasitic motion of the probe tip with respect to the test surface. Some of the uncertainty sources are temperature dependent, and several of these experience some degree of cancellation due to the partially thermally compensated design. Some uncertainties are unlikely to be improved upon and are due to manufacturing quality of various components e.g. the air bearing systems. The major components of the uncertainty are mostly temperature dependent and improvement of the temperature stability of the instrument can be used to reduce these components. Also, the thermal masses of various components, together with the anticipated cyclical nature of the temperature instability leads to some uncertainty sources which have long time constants (i.e. slowly varying) and others which may vary significantly during the measurement process.

The thermal expansion of the effective arm length is very significant because it has a short time constant (low thermal mass) compared with the total measurement cycle time of setting on the master then the test segment measurement. This error is also not directly detectable from any of the probe readings.

The horizontal thermal expansion of the machine base has a very long time constant due to the high thermal mass of the granite and steel structure. This uncertainty source is likely to be less significant.

The vertical thermal expansion of the metrology loop is partially compensated by the design and can be improved on by increasing the height (thickness) of the segment support structure (as will happen when the rotary table is replaced). Any remaining differential expansion is likely to be small and its effect is to produce a vertical probe offset (which can be nulled at the start of each measurement) and an insignificant change to the effective arm length.

The remaining uncertainty sources are then due to the manufacturing quality of the components and several of these are likely to be systematic in nature (bearing runout) and will cancel when the SAP is used in comparator mode.

7.2 Conclusions regarding P-ELT segment metrology

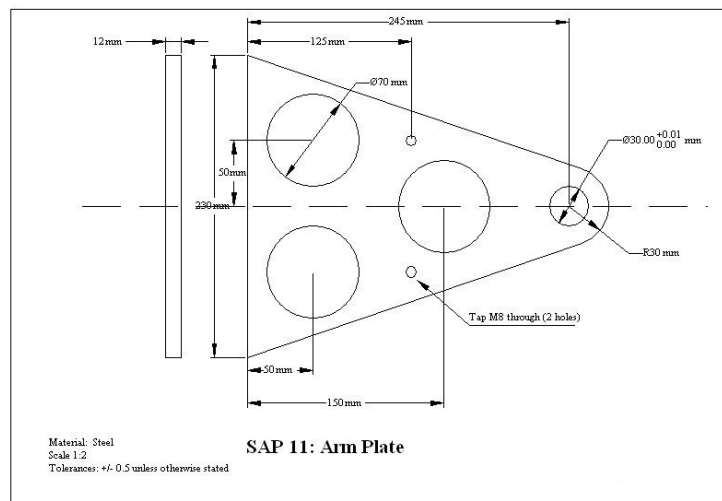
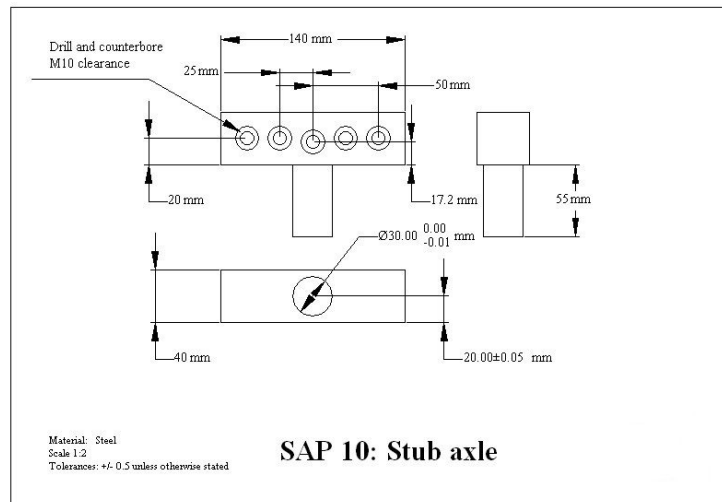
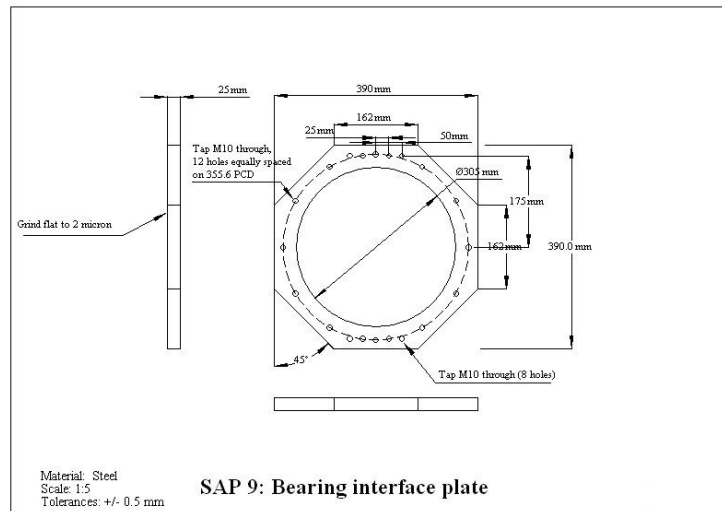
Providing that the recommendations contained in this report are implemented, the prototype swing arm profilometer, operating in comparator mode, is potentially capable of achieving uncertainties needed to provide measurement of the form error and base radius required for the ESO prototype segments, as specified in the ESO specification document [5] at §3.2.1.2, §3.2.1.3(5) and §4.3.1.6. The most critical recommendations are hereby reiterated:

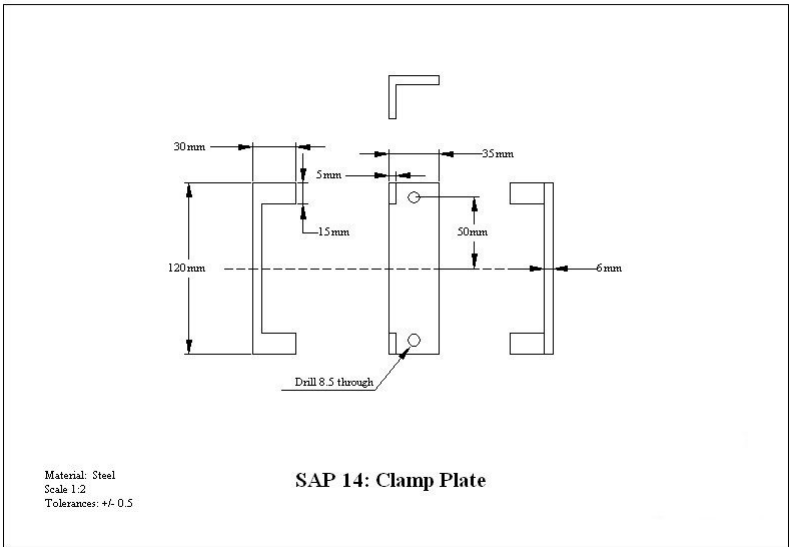
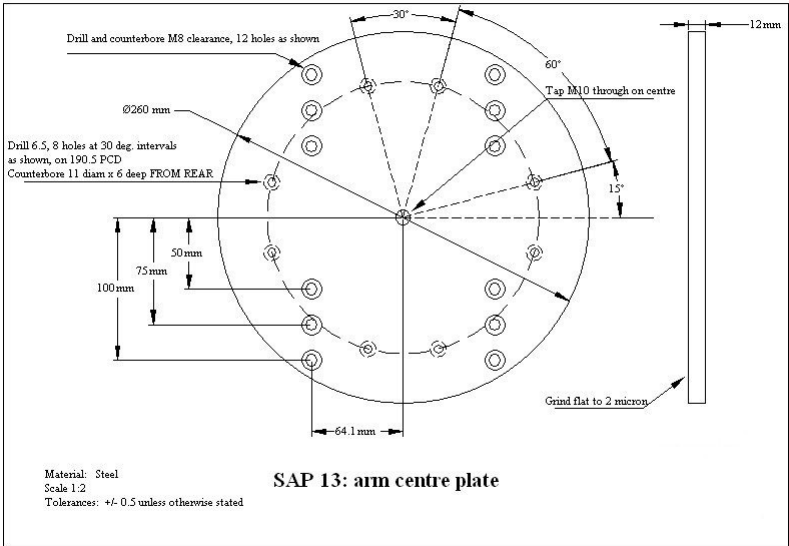
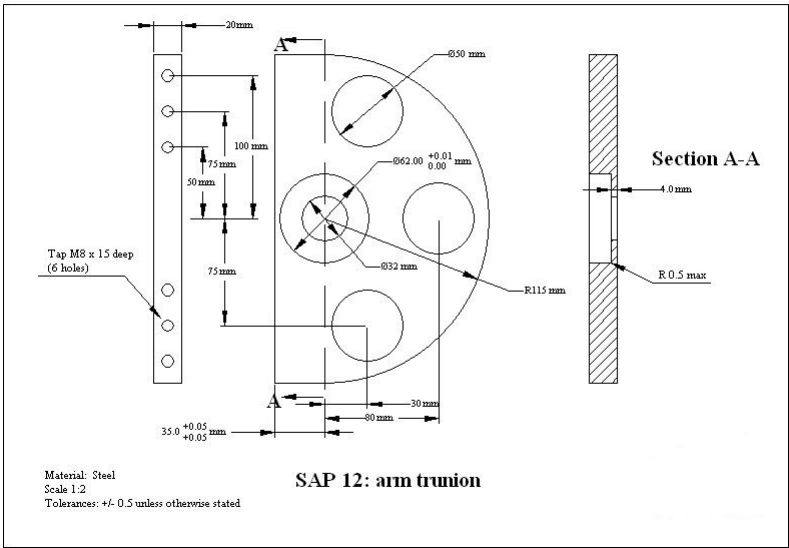
- the test is conducted differentially with respect to a master spherical segment of the same nominal base radius, manufactured to high quality;
- the main rotary table air bearing for the segment is replaced with a better model;
- the alumina ceramic arm is replaced or extended sufficiently;
- the instrument enclosure is re-assembled and the thermal control improved to ± 0.2 °C;
- the probe is replaced with a higher resolution device (shorter range allowed).

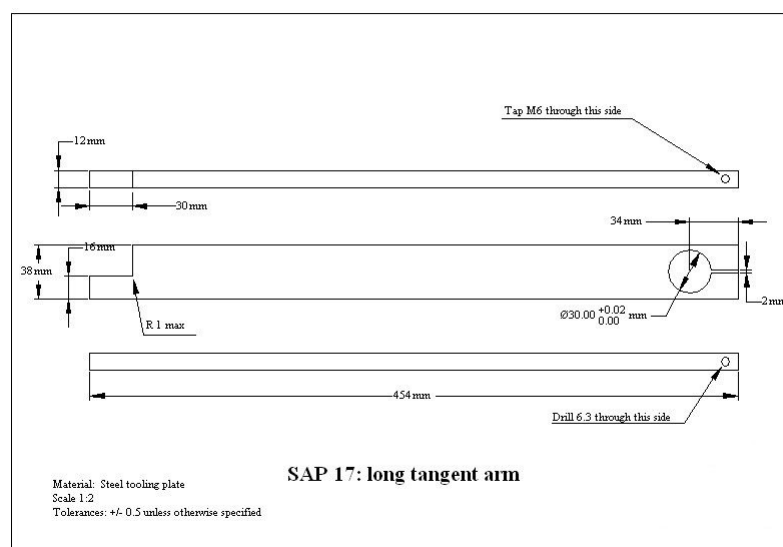
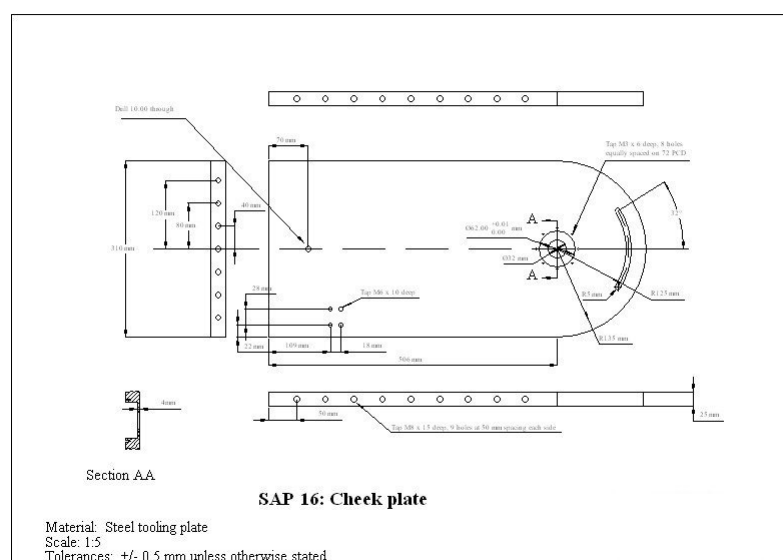
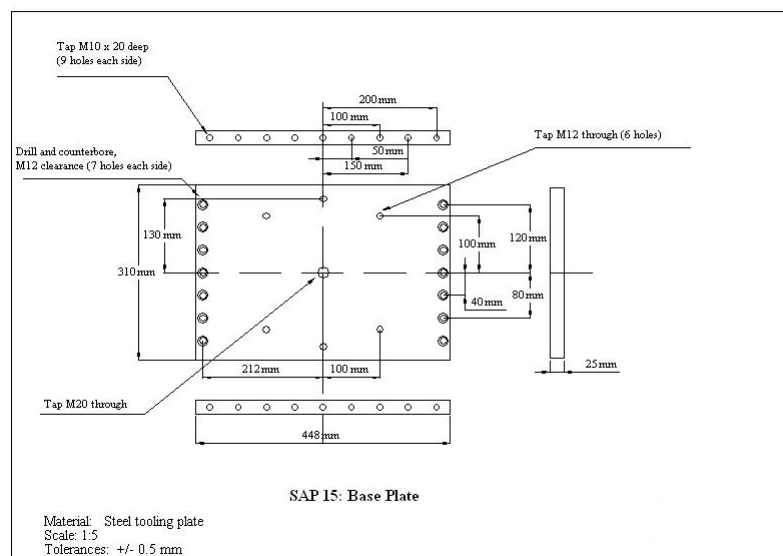
8 References

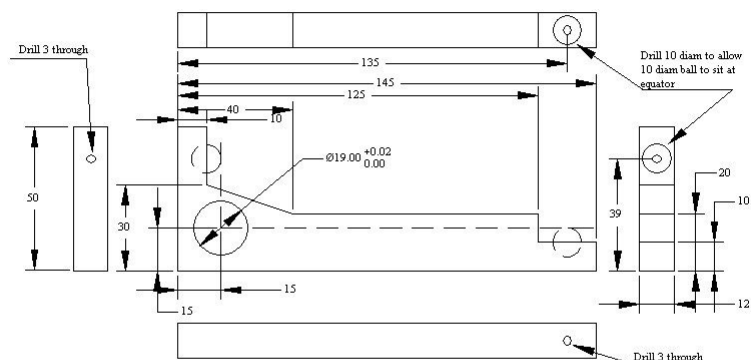
- [1] National Measurement System, Science & Technology Programme in Length Metrology, October 2002 to September 2005,
- [2] National Measurement System Programme for Engineering Measurement, October 2005 to September 2008
- [3] National Centre for Ultra-Precision Surfaces, website:
http://www.wda.co.uk/index.cfm/optic_technium/technology_centre/activities/ups/en8108
- [4] European Extremely Large Telescope (E-ELT), website:
<http://www.eso.org/public/astronomy/projects/e-elt.html>
- [5] E-ESO-SPE-300-0150, Issue 1, *Specifications for the call for tenders "Supply of 7 prototype segments for the 42 m diameter E-ELT primary mirror at a firm fixed price of €5,000,000"*
- [6] Document emailed from King C W, *Asphericity of the edge segments in E-ELT*, Eli Atad 18/02/08
- [7] Franks A, Nanometric surface metrology at the national physical laboratory, *Metrologia* **28** 1991 471-482
- [8] Leach R, Haycocks J, Jackson K, Lewis A, Oldfield S, Yacoot A, Advances in traceable nanometrology at the National Physical Laboratory, *Nanotechnology* **12** 2001 R1-R6
- [9] Peggs G N, Lewis A J, Leach R K, Measuring the metrology gap: Three-dimensional metrology at the mesoscopic level, *Journal of manufacturing processes* **6 (1)** 2004 117-124
- [10] Lewis A J, Fully traceable miniature CMM with submicrometer uncertainty, *Proc. SPIE* **5190** 265
- [11] Peggs G N, Lewis A J, Oldfield S, Design for a Compact High-Accuracy CMM, *CIRP Annals - Manufacturing Technology* **48 (1)** 1999 417-420
- [12] Knox S D, Hall S R G, Stevens R F, Traceable measurements with wavefront sensors, *Proc. SPIE* **5965** 14

Appendix A – Dimensional drawings of the main SAP components

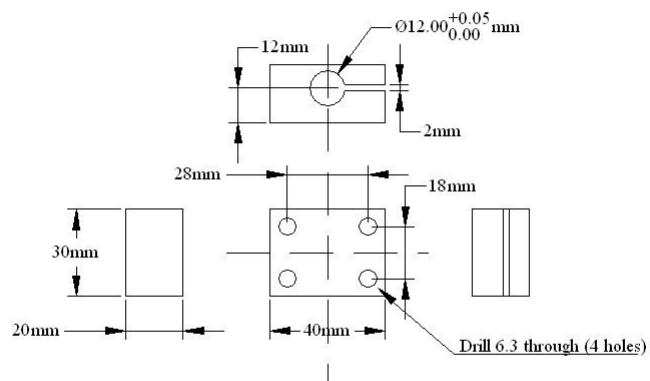




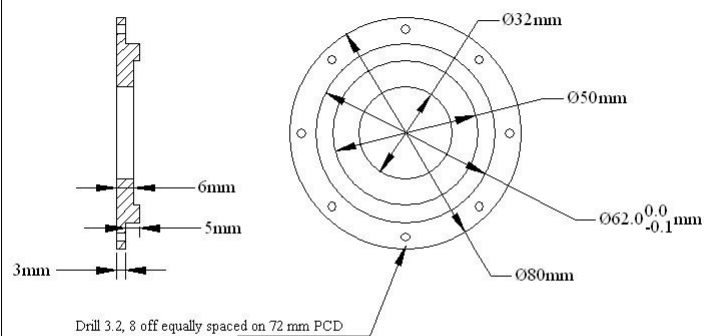


**SAP 18: Bell crank**

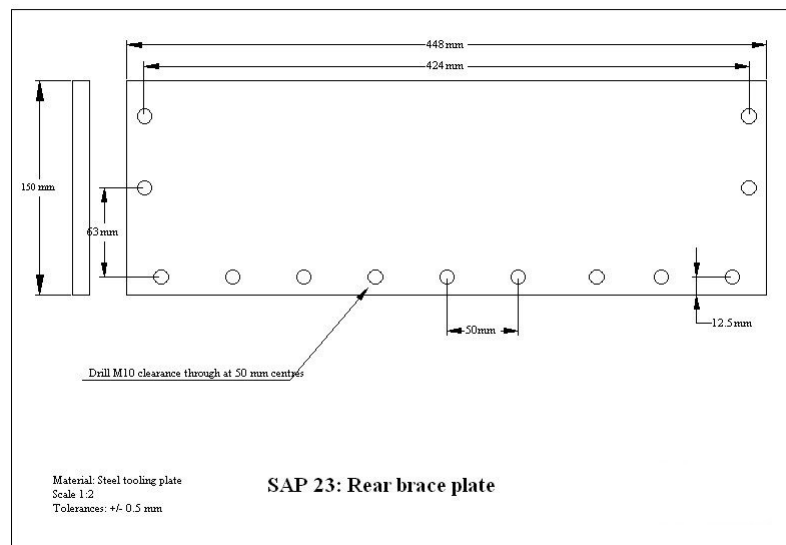
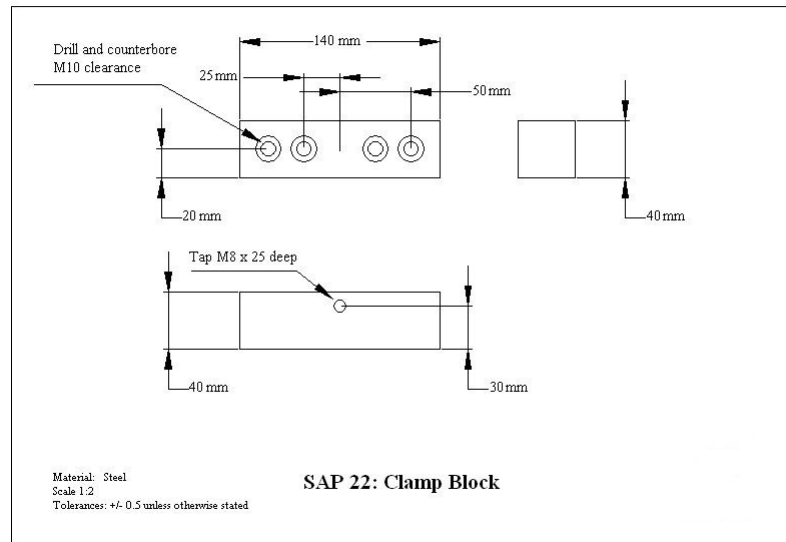
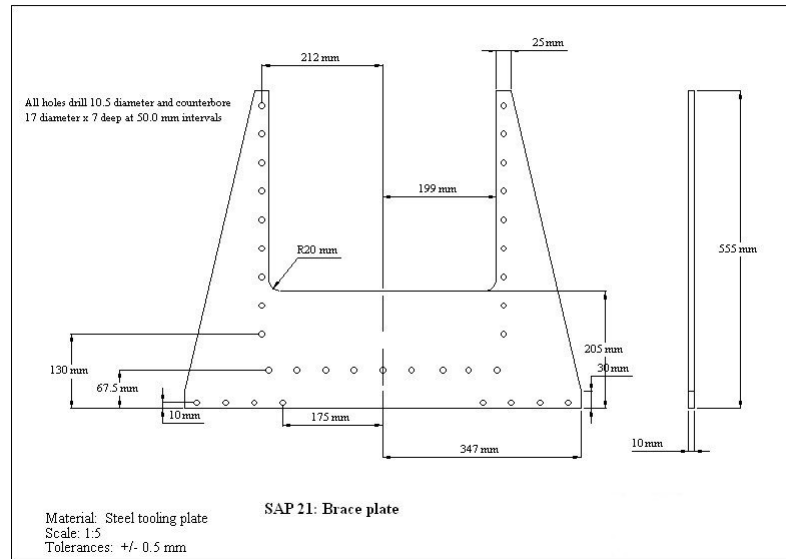
Material: Steel tooling plate
 Scale: 1:1
 Tolerances: ± 0.5 mm unless otherwise specified

**SAP 19: Micrometer block**

Material: Steel
 Scale: 1:1
 Tolerances: ± 0.5 mm unless otherwise stated

**SAP 20: Retainer**

Material: Steel
 Scale: 1:1
 Tolerances: ± 0.5 mm unless otherwise specified



Appendix B – Mathematica[®] simulation of probing errors

The modelling of a simple SAP system is based on the following analysis.

It is assumed that the reference surface is spherical and that the SAP is set up perfectly to give a zero probe reading with the probe set at the vertex of the segment. The effective arm length, L is 0.9843 m, as calculated earlier in this document, the segment base radius, R , is 84.0 m. The positional errors of the SAP with respect to the segment, ΔX , ΔY , ΔZ , are initially all zero, and the tilt angle, θ , of the SAP is set perfectly, i.e. at the angle given by

$$\sin \theta = \frac{L}{R}$$

This angle is approximately 0.671 °.

In the unperturbed coordinate system, when the SAP is set up with the probe at the segment vertex, the origin of the SAP coordinate system is deemed to be (0, 0, 0), at the point where the PI rotation axis is intersected by a normal which also runs through the probe contact point, P . The segment centre of curvature is directly above the segment vertex, at a distance of R from the vertex. P is at the vertex.

With respect to the origin of this coordinate system, the centre of curvature of the segment is located at coordinate (x , y , z) where:

$$\begin{aligned} x &= L \cos \theta \\ y &= 0 \\ z &= -L \sin \theta + R \end{aligned}$$

Now, suppose the segment stays fixed with respect to the global coordinate system, but the SAP is subject to various perturbations:

the SAP origin moves to (ΔX , ΔY , ΔZ) i.e. the SAP is displaced by ΔX , ΔY , ΔZ ;
the SAP arm length extends to $(L + \Delta L)$;
the SAP tilt angle becomes $(\theta + \Delta\theta)$.

This moves the probe tip from P to P' .

If the SAP is now allowed to sweep out a trace, whilst perturbed, then the trace will run through the usual range of sweep angle, ϕ , and at any particular value of ϕ , the probe tip location, P' , will be given, with respect to the unperturbed origin, by:

$$\begin{aligned} X' &= (L + \Delta L) \cos(\theta + \Delta\theta) \cos \phi + \Delta X \\ Y' &= (L + \Delta L) \sin \phi + \Delta Y \\ Z' &= -(L + \Delta L) \sin(\theta + \Delta\theta) \cos \phi + \Delta Z \end{aligned}$$

Therefore, the probe has been displaced from the correct location and the probe reading will be in error. Since the SAP was correctly set on the nominal base sphere and has apparently not changed, the operator would interpret the erroneous probe reading as the true shape of the segment. Thus any apparent change in the base RoC will be the error that would be observed in a real measurement.

To find the apparent probe reading, it is noted that the probe reading was set to zero for the initial correct alignment of the SAP. Since it is RoC errors that are of interest, the apparent

radius, r , of the probe point with respect to the segment centre of curvature can be calculated and compared with the nominally correct radius, which is simply R .

The distance from the perturbed probe tip, P' , to the centre of curvature is given by:

$$r = \sqrt{(X'-x)^2 + (Y'-y)^2 + (Z'-z)^2}$$

and the error in the radius measurement is simply $r - R$.

It was separately calculated that the sweep range necessary for a full trace across the segment with the SAP parameters as given above, is from -43° to $+43^\circ$.

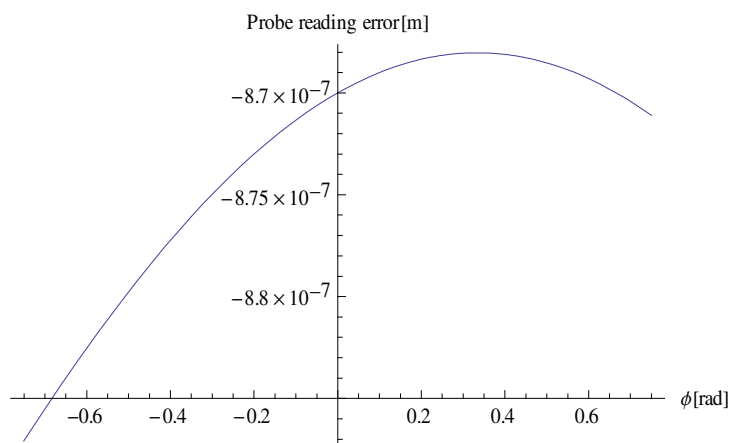
With the above parameters, it is possible to determine the magnitude and direction of the probe error, at any location during the sweep. The probe error leads, via suitable data fitting, to an error in the RoC determination.

The Mathematica code follows.

```
(*Swing Arm Error Simulation*)
(*A Lewis, June 2008*)
(*Distances are specified in metres, angles in radians*)
(*The maximum swing angle,  $\phi$ , for the E-ELT segment is  $\pm 43^\circ$ *)
 $\Delta X = 2.9 \times 10^{-6}$ ;
 $\Delta Y = 1.0 \times 10^{-6}$ ;
 $\Delta Z = 0.87 \times 10^{-6}$ ;
 $\Delta L = 0.0 \times 10^{-6}$ ;
 $L = 0.9843$ ;
 $\theta = \text{ArcSin}[L/R]$ ; (*i.e. assumes  $\theta$  is set correctly*)
 $\Delta \theta = 0.0$ ;
 $R = 84.0$ ;

 $X = (L + \Delta L) \cos[\theta + \Delta \theta] \cos[\phi] + \Delta X - L \cos[\theta]$ ;
 $Y = (L + \Delta L) \sin[\phi] + \Delta Y$ ;
 $Z = -(L + \Delta L) \sin[\theta + \Delta \theta] \cos[\phi] + \Delta Z + L \sin[\theta] - R$ ;
 $r = \sqrt{X^2 + Y^2 + Z^2}$ ;
 $b = r - R$ ;

p=Plot[b, { $\phi$ , -43 Degree, +43 Degree}, PlotRange->Automatic, AxesLabel->
{" $\phi$ [rad]", "Probe reading error[m]"}, WorkingPrecision->18]
```



```
p1=ParametricPlot3D[Evaluate[{X, Y, 1000000*b} v+{X, Y, -0.89} (1-v)],
{phi, -43 Degree, +43 Degree},{v,0,1},
BoxRatios->{1,1,1}, Mesh->None, PlotStyle->Opacity[0.7],
AxesLabel->{ "      X", "      Y", "      Error (μm)"},
AxesEdge->{{1,1},{1,1},{1,0}}]
```

