The Role of Monte Carlo Simulation in Uncertainty Evaluation

Maurice Coxy **and Bernd Sieber**tz
y **National Physical Laboratory, Teddington, Middlesex TW11 0LW, U**K_
z **Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-38116 Braunschweig, Germany**

1. Abstract

Monte Carlo Simulation (MCS) as a tool for uncertainty evaluation is considered. Advantages and disadvantages are given and contrasted with those of Mainstream GUM. A mass calibration example is presented. International guidelines being produced in the area are indicated.

2. Introduction

The role of MCS in the evaluation of measurement uncertainty is reviewed. The formulation needed to apply MCS is stated. Advantages and disadvantages of MCS are given and contrasted with those of Mainstream GUM, the “uncertainty budgeting” method summarised in Clause 8 of the Guide to the Expression of Uncertainty in Measurement (GUM) [1].

An example shows an application of MCS to mass calibration. For comparison this particular problem is solved using Mainstream GUM. It is also solved using a further approach described in the GUM that utilises “higher-order” information than provided by the model sensitivity coefficients alone.

The use of MCS for uncertainty evaluation is currently being addressed by the Joint Committee for Guides in Metrology. The intention is to prepare a supplemental guide to the GUM on the use of MCS. An indication of the likely contents of this guide is given.

3. Monte Carlo Simulation

Consider a measurement model Y = f(X), where X denotes n mutually independent input quantities and Y the measurand or output quantity.1 Numerical evaluation of the function f yields the values of Y corresponding to given values of X. The calculation might be of arbitrary complexity, ranging from the evaluation of a simple formula to the solution of a nonlinear least squares problem or partial differential equation.

Values of X are known inexactly. It is required to examine the effects on the corresponding output quantity y of changes in an estimate x of X. The “variability” of X would be quantified in any one particular situation. The best way to specify variability is by the assignment of probability distributions (pd’s) to the components $X_1; : : : ; X_n$ of X. Regardless of whether, in the language of the GUM, a Type A or Type B of evaluation of uncertainty is used, these pd’s constitute a statement of the available information concerning the input quantities. The inexactness of Y can then also be expressed in terms of a pd. As for the input quantities, this pd constitutes an assessment of the

available information concerning the measurand.
Only in simple cases can this formulation be used to obtain analytically the pd of Y. Numerical methods, based
on the following considerations, can, however, be applied. An estimate y of Y is conventionally obtained by
evaluating f for estimates (means) x_i of the input quantities X_i. However, since each X_i is described by a pd
rather than a single number, a value statistically as legitimate as its mean is given by drawing a value at random
from this pd. Each value of Y so obtained is as likely as any other.
The most effective general numerical method, MCS, [2, 5] generates a value at random from the pd for each X_i
and forms the corresponding value of Y. Repeating this process many times, to obtain in all M, say, values of Y
provides an empirical pd for Y, from which a 95% coverage interval for the measurand is obtained. The quality of
this interval depends on using an adequate value of M, advice on which and generally on implementing MCS is
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1 The more general case of correlated input quantities and more than one measurand is not considered here.