

**Uncertainty of force
generation for forces
above 1.2 MN**

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Abstract

This report details the ‘build-up’ calibration of the following NPL secondary force standard machines:

- 5 MN hydraulic amplification machine;
- 12 MN strain gauged hydraulic machine; and
- 30 MN strain gauged hydraulic machine.

The work described was performed in the period from October 1997 to September 1999, and formed deliverable 2.1.2 of Contract MPU 8/45: Mass Metrology 1996-99. This report covers the methods and equipment used to carry out the work, and analyses the results in terms of the uncertainty of force generated by each machine.

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Approved on behalf of Managing Director, NPL,
by G R Torr, Head, Centre for Mechanical and Acoustical Metrology

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UNCERTAINTY OF FORCE GENERATION FOR FORCES ABOVE 1.2 MN

1. Background

1.1 National force standards

The UK's national force standard machines are held at NPL and are used to calibrate force transducers for a wide range of customers. These transducers are then used by the customers in their own particular applications - in this way, the dissemination of traceable force standards and measurements is assured.

Most calibrations performed at NPL are in accordance with national or international procedures, and the classification obtained by the device being calibrated is partly dependent upon the uncertainty of the reference force. It is therefore evident that a lower uncertainty in the reference forces generated at NPL will provide a better facility for UK industrial customers.

The machines held and maintained at NPL generate forces either gravitationally (deadweight machines) or hydraulically.

1.2 Deadweight machines

The SI unit of force, the newton, is defined as the force required to accelerate a mass of 1 kg at a rate of $1 \text{ m} \cdot \text{s}^{-2}$. Forces generated by force standard machines must therefore be traceable to these base SI units of mass, length, and time. The most precise method currently in use to generate these forces simply involves suspending a known mass in a known gravitational field. This is the basic principle behind deadweight machines, in which a range of individual masses, or 'deadweights', is used to generate a range of discrete vertical forces.

Equation 1 gives the force F generated by a weight of conventional mass m_c and density ρ_w , in air of density ρ_a and a local gravitational acceleration of g .

$$F = m_c \cdot g \left(1 - \frac{1.2}{8000} + \frac{(1.2 - \rho_a)}{\rho_w} \right)$$

Equation 1: Vertical force generated by a known mass

Each of the terms on the right-hand side of Equation 1 can be measured to a high degree of precision, resulting in a claimed uncertainty of generated force of $\pm 0.001 \%$ for NPL's 2.5 kN, 20 kN, and 1.2 MN deadweight machines.

1.3 Hydraulic machines

NPL's largest deadweight machine has a capacity of 1.2 MN, but many customers require calibrations at forces significantly higher than this. To construct a larger deadweight machine would be economically unfeasible so NPL instead maintains a set of machines in which forces are generated hydraulically. These machines have capacities of 5 MN, 12 MN, and 30 MN.

1.3.1 5 MN machine

The 5 MN machine is based on the hydraulic amplification principle. A 22 kN deadweight machine applies a compressive force to a piston contained within a cylinder, and a hydraulic control system feeds oil into the cylinder to maintain the piston at a constant height above its rest position. This results in a stable, repeatable pressure which is fed to a much larger cylinder, lifting a piston on which the transducer to be calibrated is supported. This transducer is constrained by the machine's earth support and thus experiences a force of a magnitude given approximately by the product of the deadweight force and the area ratio of the piston / cylinder assemblies. A diagram of the machine's operation is given in Figure 1.

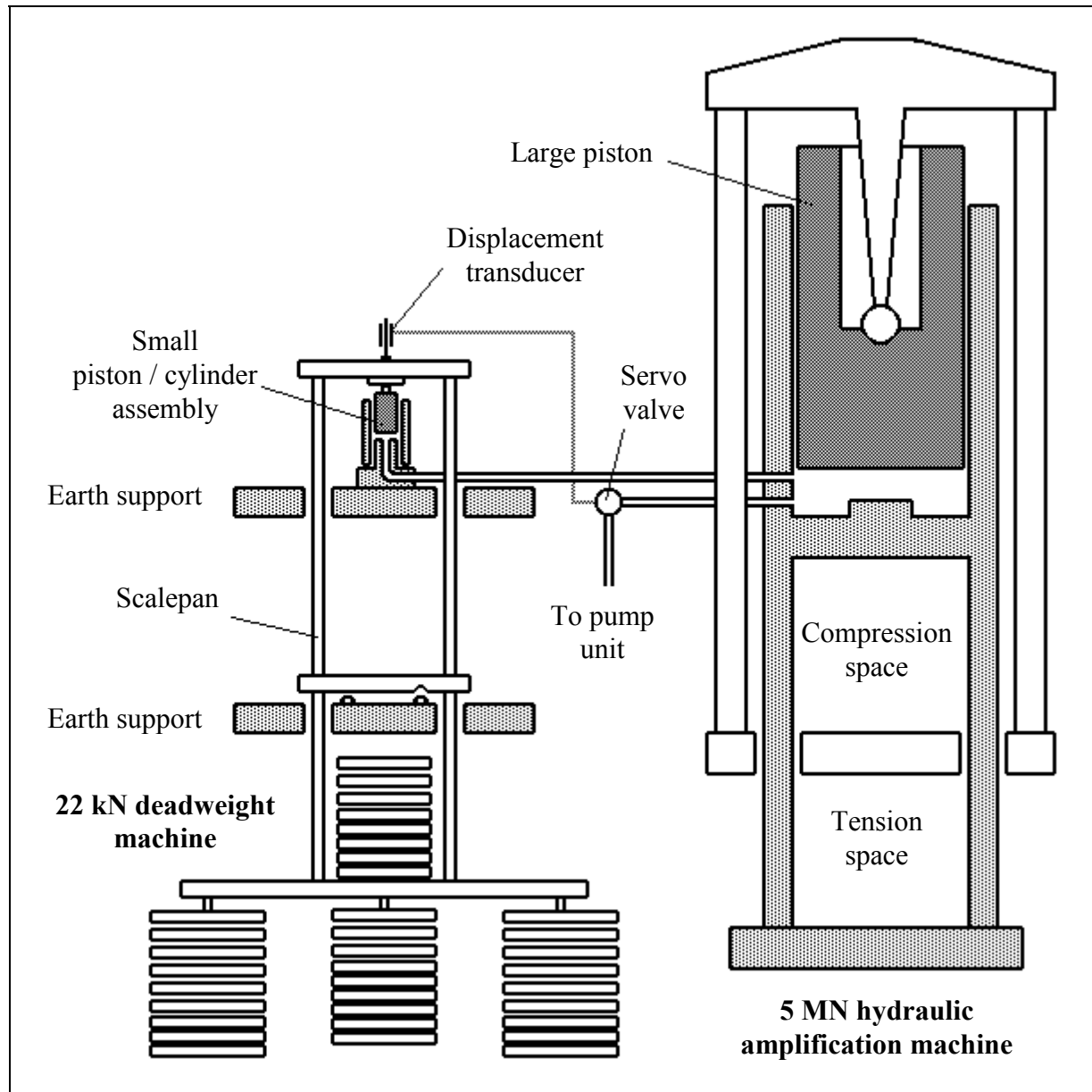


Figure 1: 5 MN hydraulic amplification machine

1.3.2 12 MN and 30 MN machines

The 12 MN and 30 MN machines work in a different manner to the 5 MN machine. An under-floor cylinder is pressurised, causing a piston to rise up. The device to be calibrated and its associated loading pads are supported on top of this piston, and the assembly travels upwards until restrained

by a horizontal loading face - this face is attached to the base of the machine by four vertical columns, each of which is strain gauged in a similar manner to a columnar load cell.

The signals from the four strain gauge bridges are displayed at the control console in units of force, the relationship between the applied force and the bridge outputs having been determined during the calibration of the machine.

1.4 Calibration of hydraulic machines

Calibration of the hydraulic machines is defined as the process by which, at given nominal forces, the actual forces being applied to a proving device are determined. This process can either be indirect, in which the applied forces are calculated, or direct, in which the applied forces are measured.

1.4.1 Indirect calibration

The forces applied to proving devices in the 5 MN machine can be calculated from the deadweight force and the amplification ratio, making allowances for: the reliance of this ratio on temperature and pressure; the pressure drops within the hydraulic circuitry; the hysteresis of the piston / cylinder assemblies; the vertical alignment of the piston / cylinder assemblies; the non-axial rotation of the smaller piston / cylinder assembly; and various other minor effects.

Similarly, the forces applied in the 12 MN and 30 MN machines could be calculated from the strain gauge bridge outputs, using knowledge of: the gauge factors of the strain gauges; the material properties of the columns; the dimensions of the columns; the strain distribution within the columns; the temperature sensitivity of the measuring system; and other effects.

The disadvantage of these indirect calibration methods is precisely that - they are indirect and thus do not measure the actual force. If terms are incorrectly estimated or calculated, or omitted altogether, the result can be forces quoted to a very small uncertainty which are grossly in error. Also, for the strain gauged machines, the precision with which the applied force could be calculated from the various parameters would not be sufficiently good for their purpose.

1.4.2 Direct calibration

The process by which hydraulic machines are directly calibrated is known as the 'build-up' procedure. At NPL, three load cells are calibrated individually in the 1.2 MN deadweight machine, and then loaded in parallel in the 5 MN machine, giving a traceable force measurement up to 3.6 MN. The 5 MN machine is then used to calibrate three load cells up to 3.6 MN individually - these cells are then used in parallel to calibrate the 5 MN machine up to 5 MN and the 12 MN and 30 MN machines up to 10.8 MN. A traceable force capability is thus 'built up' from the capacity of the largest deadweight machine.

NPL prefer to use the direct, 'build-up' method because it does measure the actual forces applied to proving devices, and it requires neither detailed design knowledge nor assumptions to be made about the performance of the machines.

1.5 Changes from previous 'build-up' exercises

A number of changes from previous work were introduced for the present 'build-up' exercise. These can be summarised as follows:

- all of the data acquisition was performed automatically, using custom-written software to communicate via an IEEE link between a laptop PC and a precision AC ratio meter - this computer control allowed strict, repeatable timings to be adhered to;
- the data analysis was performed in generic spreadsheets, allowing uncertainty calculations to be performed quickly and efficiently; and
- the relative orientation of the load cells was fine-tuned by comparing the forces seen by each of the three, and adjusting the positions until all were seeing approximately the same force - this ensured the best alignment possible within the machine.

2. Method

Full details of the ‘build-up’ procedure are contained within NPL procedure QPMAM/B/283, but a summary of the salient points is given here.

2.1 Equipment

Various sets of load cells are used for the work - these comprise three of 1.2 MN capacity, three of 4.5 MN capacity, and three of 10 MN capacity. They are loaded either individually or in parallel with the others of the same capacity, and energised by a precision AC ratio meter. This instrument is connected to a notebook computer which runs the software and takes readings automatically.

2.2 Setting up

All calibrations are carried out at a laboratory temperature of $20\text{ }^{\circ}\text{C} \pm 1\text{ }^{\circ}\text{C}$, with all instrumentation left energised for at least 12 hours prior to the work.

The load cells are centred on the machine’s compression platen to within 0.5 mm. When three load cells are loaded in parallel, the force measured by each is checked by the software, and a fine adjustment of the relative positions may then be carried out to spread the load more evenly.

2.3 Calibration runs

Prior to the first calibration run, a preload equal to the maximum calibration force is applied six times. This is repeated at the start of each day and carried out three times after each load cell rotation.

It is important that repeatable timings are adhered to, so a standard dwell period of two minutes is specified - this is the period after the force has been changed before any readings are taken, both in the calibration runs and during the preload sequences.

Each full calibration consists of seven calibration runs, with both incremental and decremental forces applied. The specific forces used are chosen to give a comprehensive coverage of the force range. It is important that the same forces are applied in the load cells’ subsequent usages, in order that the timing integrity be maintained.

2.4 5 MN machine

This machine is calibrated using a single load cell up to 1.2 MN and all three 1.2 MN load cells in parallel up to 3.6 MN. Three 4.5 MN load cells are then individually calibrated in the machine up to 3.6 MN - they are then used in parallel to calibrate the machine up to 5.0 MN.

2.5 12 MN machine

This machine is calibrated using the three 1.2 MN load cells up to 3.6 MN, and then the three 4.5 MN load cells up to 10.8 MN.

2.6 30 MN machine

This machine is calibrated using the three 1.2 MN load cells up to 3.6 MN, the three 4.5 MN load cells up to 10.8 MN, and then the three 10 MN load cells (each having been calibrated individually in the 30 MN machine) up to 30 MN.

3. Results

Only the final results for each machine are summarised here, except for the 5 MN machine, for which example results and calculations are given in detail. It should be noted that, for all machines, a significant amount of work was carried out prior to the final work which resulted in the following values, as the optimisation of the machines' performance is an iterative process.

3.1 5 MN machine

Each of the three 1.2 MN load cells was calibrated individually in the 1.2 MN deadweight machine. The 5 MN machine was then calibrated from 100 kN to 1.2 MN using one of the cells, and from 300 kN to 3.6 MN using all three in parallel. The results obtained from the three load cells in parallel are given in Table 1. The readings have been corrected to give deflections by removing the output at zero load, assuming a linear drift in this zero output throughout each calibration run.

Table 1: 5 MN machine individual deflections - 300 kN to 3.6 MN

Force / kN	Run 1			Run 2			Run 3		
	1200/1C	1200/2C	1200/3C	1200/1C	1200/2C	1200/3C	1200/1C	1200/2C	1200/3C
300	0.175 437	0.186 535	0.180 580	0.175 165	0.187 656	0.179 741	0.177 424	0.188 387	0.176 785
450	0.263 928	0.279 373	0.269 983	0.264 068	0.280 495	0.268 554	0.266 403	0.281 341	0.265 463
600	0.353 266	0.370 544	0.360 328	0.353 808	0.371 784	0.358 582	0.356 385	0.372 111	0.355 725
900	0.532 127	0.551 679	0.541 510	0.533 400	0.552 661	0.539 367	0.536 954	0.550 800	0.537 917
1 200	0.710 861	0.733 036	0.722 593	0.712 572	0.734 054	0.719 962	0.717 482	0.729 232	0.719 816
1 800	1.067 977	1.095 543	1.083 771	1.070 463	1.096 249	1.080 691	1.079 055	1.085 095	1.083 347
2 400	1.426 066	1.456 383	1.444 074	1.428 569	1.457 844	1.440 373	1.440 388	1.441 062	1.445 516
3 000	1.784 218	1.816 782	1.803 208	1.786 567	1.818 765	1.799 198	1.802 027	1.795 396	1.807 208
3 600	2.140 516	2.177 439	2.162 121	2.143 101	2.180 379	2.157 130	2.163 161	2.150 069	2.167 351
3 000	1.792 275	1.815 021	1.797 261	1.781 253	1.811 933	1.811 518	1.811 335	1.792 972	1.800 679
2 400	1.428 801	1.448 693	1.449 870	1.429 009	1.449 947	1.448 634	1.443 780	1.432 998	1.450 834
1 800	1.069 997	1.087 741	1.090 642	1.070 079	1.087 553	1.090 578	1.081 235	1.072 226	1.094 915
1 200	0.714 795	0.724 589	0.727 970	0.713 672	0.724 484	0.729 430	0.721 352	0.711 298	0.734 996
900	0.536 090	0.543 211	0.547 169	0.535 030	0.543 018	0.548 444	0.541 729	0.530 881	0.554 004
600	0.356 737	0.361 776	0.366 299	0.355 989	0.361 341	0.367 594	0.361 313	0.350 699	0.372 783
450	0.267 034	0.270 982	0.275 734	0.266 502	0.270 195	0.277 235	0.271 246	0.260 472	0.282 060
300	0.176 701	0.179 794	0.185 994	0.177 164	0.178 521	0.187 028	0.181 771	0.167 424	0.193 681

Force / kN	Run 4			Run 5			Run 6		
	1200/1C	1200/2C	1200/3C	1200/1C	1200/2C	1200/3C	1200/1C	1200/2C	1200/3C
300	0.177 082	0.190 681	0.174 707	0.179 381	0.193 935	0.169 172	0.180 460	0.193 032	0.168 938
450	0.265 983	0.285 382	0.261 908	0.268 882	0.289 340	0.254 886	0.270 268	0.289 576	0.253 493
600	0.354 956	0.377 600	0.351 674	0.360 252	0.380 059	0.343 868	0.360 052	0.382 849	0.341 212
900	0.535 502	0.555 990	0.534 149	0.541 528	0.557 594	0.526 405	0.542 469	0.558 891	0.524 132
1 200	0.715 563	0.734 115	0.716 962	0.723 731	0.733 867	0.708 917	0.724 471	0.733 234	0.708 980
1 800	1.077 076	1.089 535	1.080 976	1.088 298	1.086 617	1.072 466	1.089 284	1.084 884	1.073 237
2 400	1.438 200	1.445 616	1.443 280	1.455 839	1.431 993	1.439 072	1.454 856	1.435 871	1.436 292
3 000	1.799 579	1.800 329	1.804 807	1.820 673	1.782 808	1.801 260	1.820 416	1.785 794	1.798 481
3 600	2.160 322	2.155 810	2.164 335	2.185 840	2.129 525	2.165 063	2.186 082	2.135 520	2.158 928
3 000	1.808 683	1.798 293	1.798 190	1.829 794	1.778 099	1.797 234	1.829 575	1.783 657	1.791 660
2 400	1.439 649	1.442 520	1.445 678	1.456 078	1.420 374	1.451 144	1.455 057	1.426 983	1.445 714
1 800	1.077 009	1.080 467	1.091 322	1.087 418	1.058 749	1.102 464	1.084 734	1.066 002	1.097 935
1 200	0.718 710	0.717 463	0.731 613	0.726 767	0.700 575	0.740 444	0.724 311	0.706 382	0.736 975
900	0.539 416	0.536 394	0.550 678	0.546 335	0.521 910	0.558 353	0.544 113	0.527 604	0.554 913
600	0.359 796	0.355 390	0.369 815	0.365 359	0.343 788	0.375 847	0.363 292	0.348 722	0.373 050
450	0.270 158	0.264 500	0.279 156	0.274 640	0.254 647	0.284 640	0.273 091	0.259 169	0.281 715
300	0.181 524	0.170 247	0.191 041	0.184 562	0.163 096	0.195 384	0.183 009	0.166 794	0.193 171

Force / kN	Run 7		
	1200/1C	1200/2C	1200/3C
300	0.178 910	0.190 139	0.173 267
450	0.268 707	0.282 372	0.262 153
600	0.357 501	0.376 304	0.350 118
900	0.539 594	0.548 956	0.536 800
1 200	0.720 855	0.724 179	0.721 376
1 800	1.083 715	1.074 052	1.089 684
2 400	1.446 854	1.422 311	1.457 430
3 000	1.809 994	1.768 949	1.825 377
3 600	2.172 626	2.115 733	2.192 008
3 000	1.818 917	1.763 533	1.822 205
2 400	1.452 634	1.404 005	1.470 728
1 800	1.090 826	1.042 812	1.114 607
1 200	0.730 889	0.687 915	0.748 570
900	0.550 039	0.511 789	0.564 516
600	0.368 313	0.335 979	0.380 190
450	0.277 457	0.248 058	0.288 261
300	0.186 680	0.158 036	0.197 807

For each load cell, the deflections obtained at each force were averaged. Second order polynomial fits applied to the calibration data from the 1.2 MN machine gave a calibration factor at each measured force for each load cell - these calibration factors were used to convert the measured voltage ratio deflections to force values. The force values from each of the three load cells were then summed to give the total force applied. The error associated with each nominal force value was then able to be estimated. The results of this analysis are given in Table 2.

Table 2: 5 MN machine mean deflections, resultant forces, and calculated errors - 300 kN to 3.6 MN

Force kN	Mean deflection / $\text{mV}\cdot\text{V}^{-1}$			Force / kN			Total kN	Error kN	Error %
	1200/1C	1200/2C	1200/3C	1200/1C	1200/2C	1200/3C			
300	0.177 846	0.190 063	0.174 557	98.45	105.18	96.49	300.12	0.12	0.040
450	0.267 118	0.283 781	0.262 324	147.89	157.08	145.04	450.00	0.00	0.001
600	0.356 715	0.375 944	0.351 453	197.53	208.13	194.36	600.02	0.02	0.003
900	0.537 646	0.553 191	0.534 635	297.84	306.38	295.77	899.99	-0.01	-0.001
1 200	0.718 299	0.730 737	0.717 498	398.07	404.87	397.09	1 200.03	0.03	0.003
1 800	1.079 948	1.085 753	1.081 732	598.96	602.05	599.14	1 800.14	0.14	0.008
2 400	1.442 203	1.439 174	1.445 433	800.51	798.64	801.21	2 400.36	0.36	0.015
3 000	1.804 184	1.792 221	1.808 114	1 002.23	995.33	1 003.04	3 000.60	0.60	0.020
3 600	2.165 534	2.145 026	2.169 868	1 203.93	1 192.18	1 204.68	3 600.79	0.79	0.022
3 000	1.811 344	1.788 380	1.805 119	1 006.19	993.15	1 001.34	3 000.68	0.68	0.023
2 400	1.444 705	1.428 691	1.454 166	801.81	792.71	805.97	2 400.50	0.50	0.021
1 800	1.081 516	1.067 295	1.099 634	599.71	591.64	608.94	1 800.29	0.29	0.016
1 200	0.722 673	0.707 578	0.737 321	400.36	391.86	407.93	1 200.15	0.15	0.013
900	0.542 849	0.528 325	0.555 324	300.60	292.44	307.10	900.14	0.14	0.016
600	0.362 389	0.349 209	0.373 221	200.58	193.20	206.29	600.08	0.08	0.013
450	0.272 198	0.259 510	0.282 133	150.62	143.54	155.91	450.07	0.07	0.016
300	0.182 261	0.167 743	0.192 739	100.83	92.76	106.48	300.07	0.07	0.025

In order to calculate the uncertainty associated with each of these error estimates, it is necessary to calculate both the repeatability and reproducibility of the calibration results. The repeatability is defined as the stability of the measured force during a particular force application. At each force during each run, the repeatability is calculated from Equation 2 (where s_i are the five consecutive readings taken at each force), and then all repeatability values obtained at each force are averaged to give a mean repeatability.

$$\text{repeatability} = u_{\text{repeat}} = \left[\frac{\sigma_{n-1}(s_i)}{\sqrt{5}} \right]$$

Equation 2: Repeatability

Reproducibility is defined as the stability of measured force from run to run and is calculated in Equation 3 (where s_j are the total measured forces in the seven runs).

$$\text{reproducibility} = u_{\text{repro}} = \left[\frac{\sigma_{n-1}(s_j)}{\sqrt{7}} \right]$$

Equation 3: Reproducibility

The data have also been analysed to give the measured error at each nominal force in each run - these values are given in Table 3, together with repeatability and reproducibility values, expressed in percentage terms.

Table 3: 5 MN machine measured errors, repeatability, and reproducibility - 300 kN to 3.6 MN

Force kN	Measured errors / %							Repeatability %	Reproducibility %
	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7		
300	0.054	0.056	0.063	0.040	0.045	0.034	0.012	0.006	0.004
450	0.007	-0.013	-0.002	0.007	-0.012	0.016	0.002	0.001	0.004
600	0.004	0.008	0.013	0.014	0.010	0.004	-0.014	0.001	0.002
900	-0.011	-0.004	0.011	0.009	0.003	0.001	-0.009	0.001	0.003
1 200	0.000	0.005	0.002	0.007	0.002	0.010	-0.003	0.001	0.001
1 800	0.003	0.007	0.010	0.013	0.007	0.007	0.008	0.001	0.001
2 400	0.008	0.014	0.018	0.022	0.017	0.020	0.010	0.001	0.002
3 000	0.014	0.020	0.022	0.024	0.024	0.023	0.016	0.001	0.001
3 600	0.016	0.025	0.024	0.022	0.022	0.024	0.021	0.001	0.001
3 000	0.018	0.020	0.026	0.029	0.028	0.024	0.019	0.001	0.002
2 400	0.016	0.021	0.022	0.027	0.022	0.025	0.016	0.001	0.001
1 800	0.014	0.009	0.014	0.027	0.022	0.023	0.009	0.001	0.003
1 200	0.003	0.014	0.016	0.023	0.023	0.017	0.003	0.001	0.003
900	0.015	0.016	0.023	0.015	0.022	0.024	0.006	0.001	0.002
600	0.012	0.023	0.010	0.029	0.029	0.035	-0.019	0.002	0.004
450	0.005	0.027	0.008	0.013	0.026	0.033	0.008	0.001	0.004
300	-0.022	0.019	0.049	0.037	0.079	0.067	-0.017	0.001	0.014

The repeatability and reproducibility can be added in quadrature to give a combined uncertainty. However, these values are the repeatability and reproducibility of the forces measured during the calibration and not those solely due to the machine, as the load cells themselves are not perfect. As the aim of the exercise is to determine the performance of the machine alone, it is necessary to remove the effects of the load cells from these figures. This can only be done if the uncertainty of the measured force is greater than the uncertainty associated with the load cell at that force. If this is not the case, the uncertainty of generated force is taken to be that of the load cell itself.

Once the load cell contributions have been extracted and the combined uncertainty calculated, the effective degrees of freedom (v_{eff} , from Equation 4) can be determined and the 95 % confidence limits can be calculated using the coverage factor k_{95} given in Table 4. These figures are given in Table 5, and the error / uncertainty profiles of the machine are plotted in Figure 2 and Figure 3 for incremental and decremental forces respectively.

$$v_{eff} = (\text{combined uncertainty})^4 \left/ \left(\frac{\text{reproducibility}^4}{6} + \frac{\text{repeatability}^4}{34} \right) \right.$$

Equation 4: simplified Welch-Satterthwaite equation

Table 4: Coverage factors for various degrees of freedom

v_{eff}	1	2	3	4	5	6	7	8	10	12	14	16
k_{95}	13.97	4.53	3.31	2.87	2.65	2.52	2.43	2.37	2.28	2.23	2.20	2.17

v_{eff}	18	20	25	30	35	40	45	50	60	80	100	∞
k_{95}	2.15	2.13	2.11	2.09	2.07	2.06	2.06	2.05	2.04	2.03	2.02	2.00

Table 5: 5 MN machine uncertainty figures - 300 kN to 3.6 MN

Force kN	Repeatability		Reproducibility		Combined uncertainty	v_{eff}	k_{95}	95 % uncertainty	min. error	max. error
	Cell %	Machine %	Cell %	Machine %	%			%	%	%
300	0.001	0.005	0.007	0.004	0.007	30.57	2.12	0.014	0.025	0.054
450	0.000	0.001	0.005	0.004	0.005	7.20	2.42	0.011	-0.010	0.012
600	0.000	0.001	0.003	0.002	0.002	17.53	2.15	0.004	-0.001	0.007
900	0.000	0.001	0.007	0.003	0.003	6.88	2.44	0.008	-0.009	0.007
1 200	0.000	0.001	0.008	0.001	0.002	11.96	2.22	0.004	-0.001	0.006
1 800	0.000	0.001	0.005	0.001	0.001	11.38	2.24	0.003	0.005	0.011
2 400	0.000	0.001	0.004	0.002	0.002	13.28	2.20	0.005	0.010	0.020
3 000	0.000	0.001	0.004	0.001	0.002	18.19	2.14	0.004	0.016	0.024
3 600	0.000	0.001	0.004	0.001	0.002	19.70	2.13	0.003	0.018	0.025
3 000	0.000	0.001	0.005	0.002	0.002	9.86	2.28	0.004	0.018	0.027
2 400	0.000	0.001	0.006	0.001	0.002	9.49	2.29	0.004	0.017	0.024
1 800	0.000	0.001	0.008	0.003	0.003	7.29	2.41	0.007	0.009	0.022
1 200	0.000	0.001	0.009	0.003	0.003	8.08	2.36	0.007	0.006	0.020
900	0.000	0.001	0.011	0.002	0.002	18.60	2.14	0.005	0.011	0.021
600	0.000	0.002	0.010	0.004	0.004	9.43	2.30	0.010	0.003	0.022
450	0.000	0.001	0.009	0.004	0.005	7.31	2.41	0.011	0.005	0.027
300	0.000	0.001	0.011	0.008	0.008	6.25	2.50	0.020	0.004	0.045

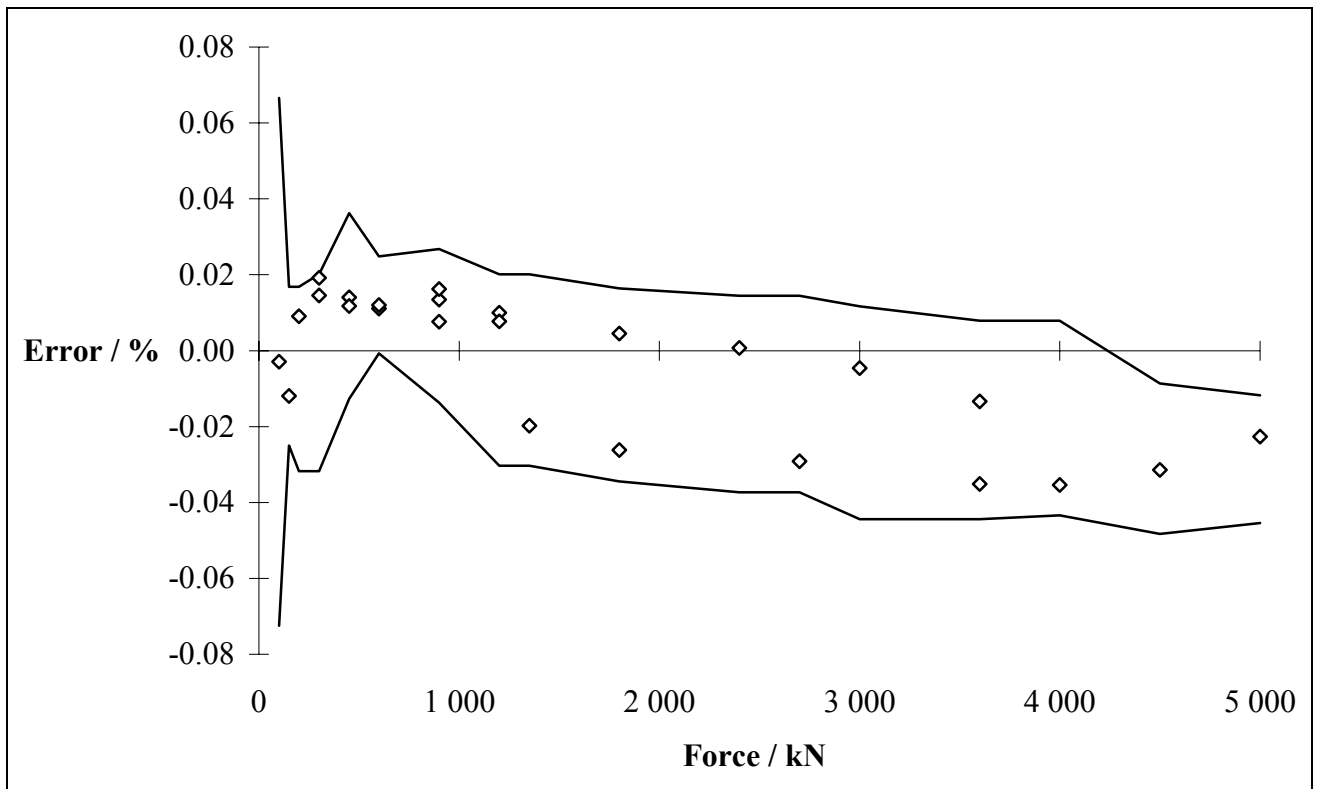


Figure 2: Incremental errors of 5 MN machine forces, with 95 % confidence intervals

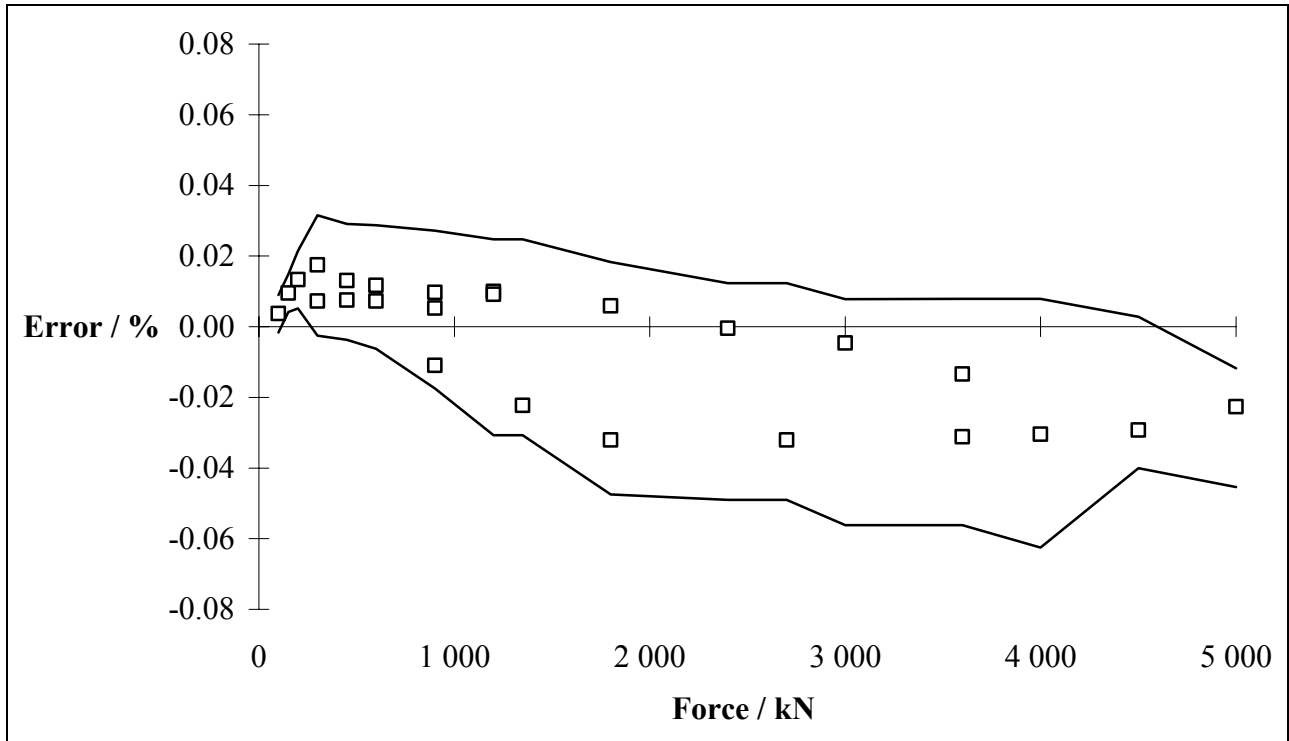


Figure 3: Decremental errors of 5 MN machine forces, with 95 % confidence intervals

The two preceding figures demonstrate that, if systematic corrections are made to the forces generated by the machine (by adding or subtracting weights to or from the scalepan of the

deadweight machine), it is possible to generate the nominal forces required with an uncertainty of less than 0.05 % over a large proportion of the machine's range. Figure 4 and Figure 5 demonstrate this post-correction performance.

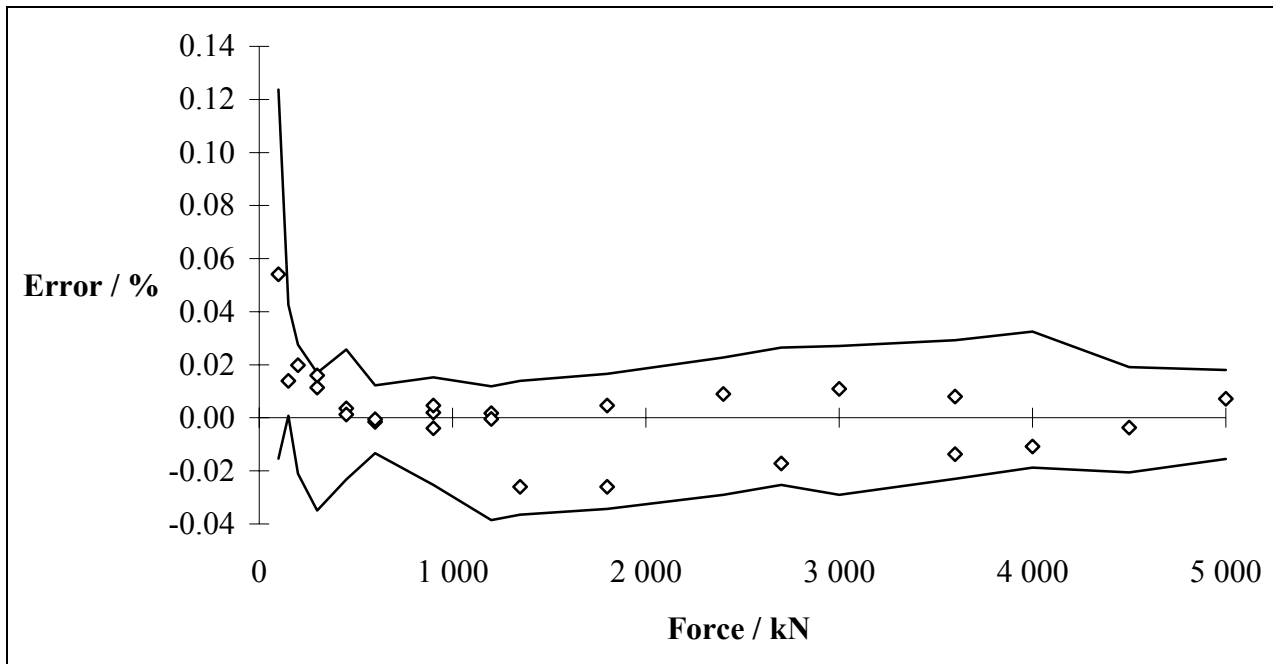


Figure 4: Post-correction incremental errors of 5 MN machine forces, with 95 % confidence intervals

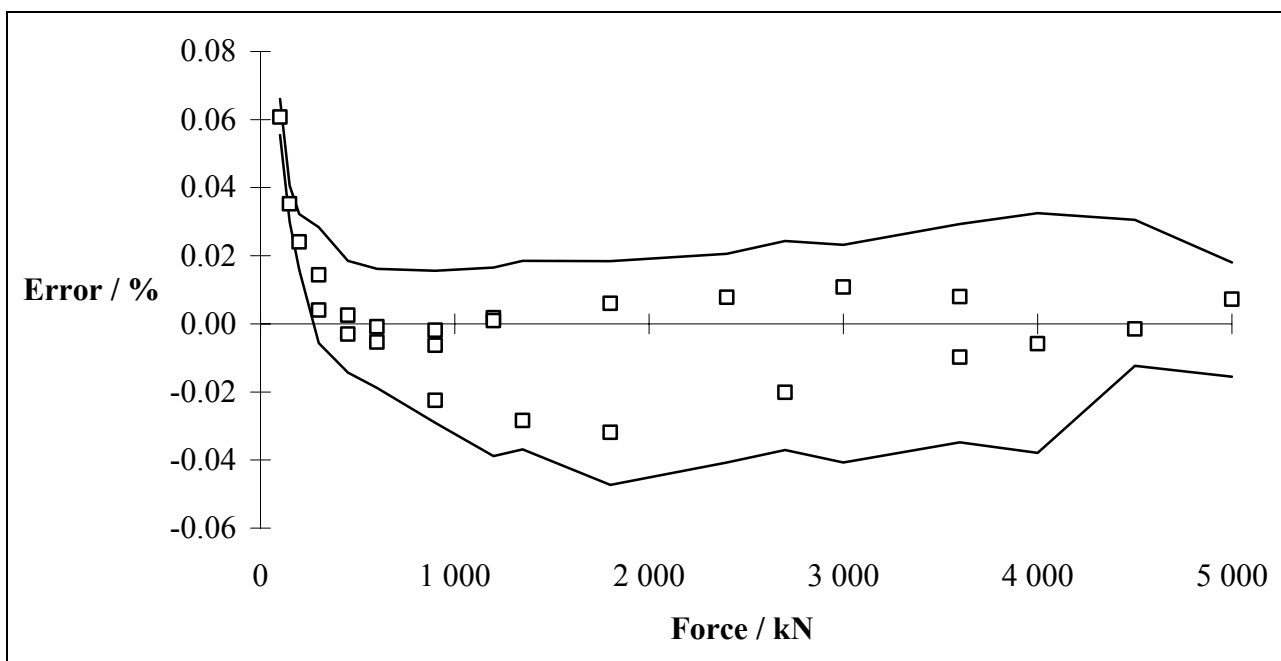


Figure 5: Post-correction decremental errors of 5 MN machine forces, with 95 % confidence intervals

3.2 12 MN machine

The calibration results obtained in the 12 MN machine, for the two sets of load cells used, are shown in Figure 6 and Figure 7 for incremental and decremental forces respectively.

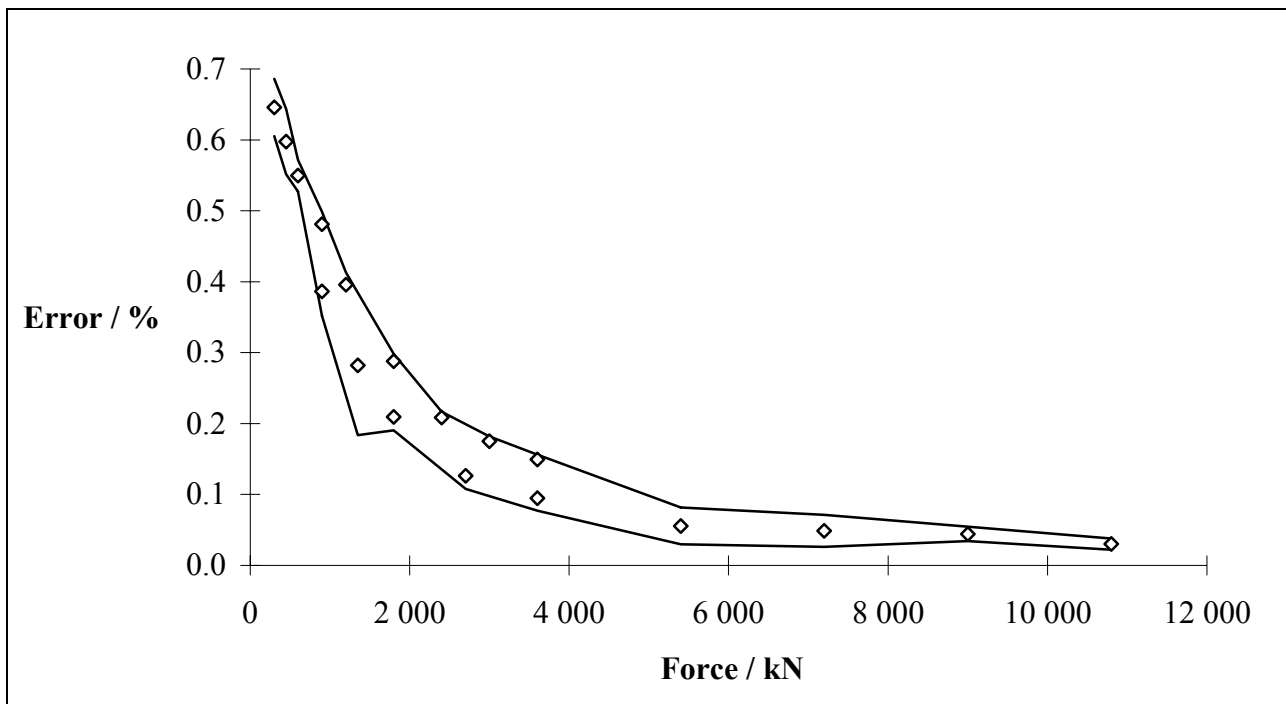


Figure 6: Incremental errors of 12 MN machine forces, with 95 % confidence intervals

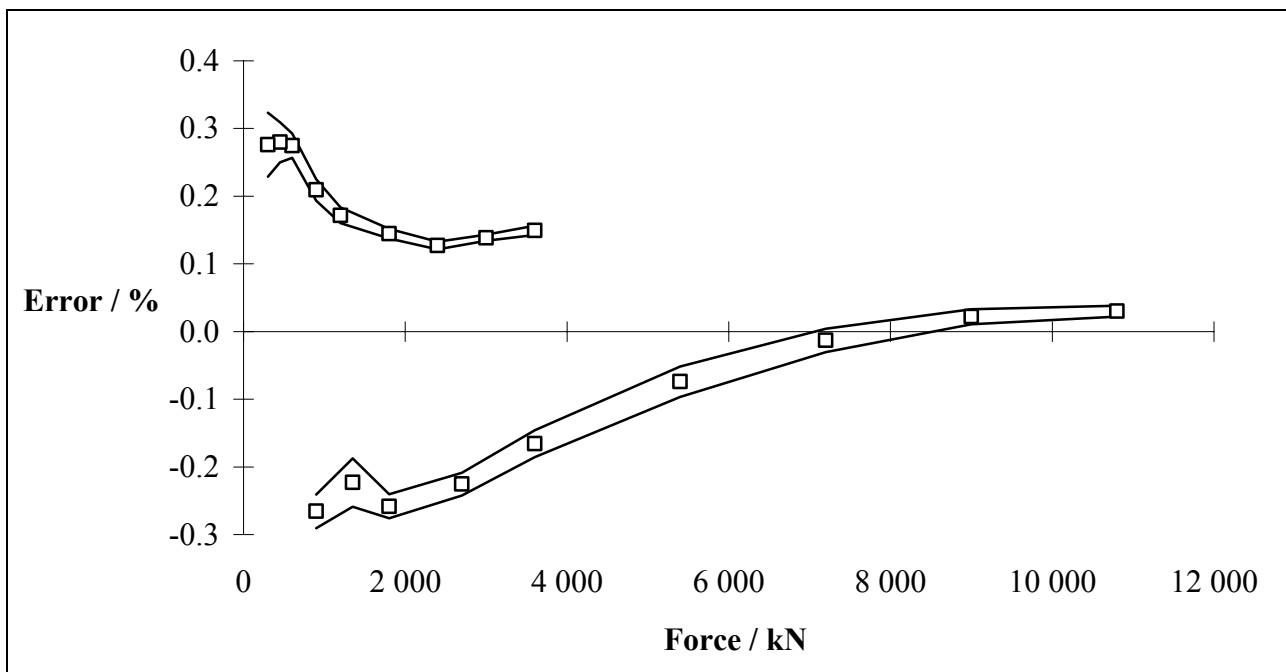


Figure 7: Decremental errors of 12 MN machine forces, with 95 % confidence intervals

The two main conclusions drawn from the two preceding figures were:

- the estimated error at the lower incremental forces was unacceptably large;
- the machine displayed a large amount of hysteresis, leading to decremental errors significantly dependent upon the maximum force applied.

The machine's indicator was adjusted, based on the calibration results, to bring the incremental forces closer to the required values, and the post-adjustment calibration results are given in Figure 8.

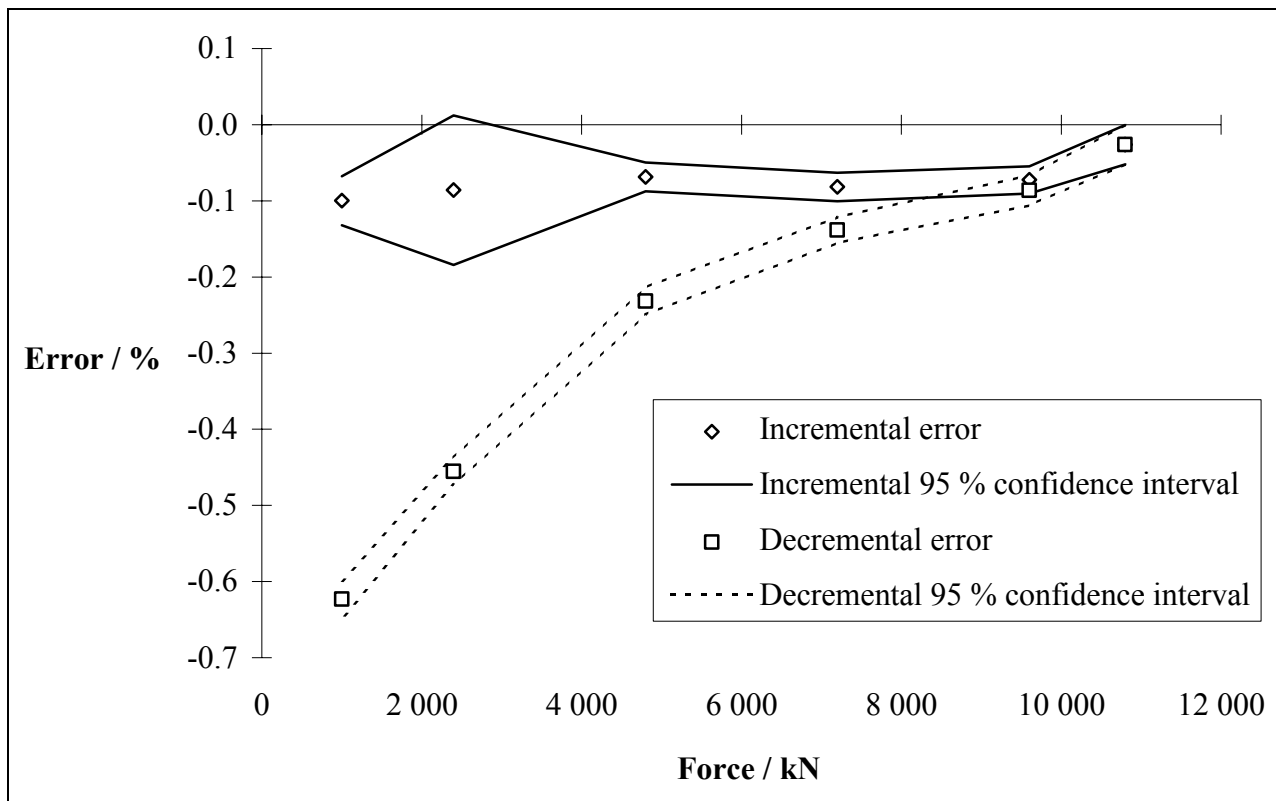


Figure 8: 12 MN machine re-calibration results

3.3 30 MN machine

The calibration results obtained in the 30 MN machine, for the three sets of load cells used, are shown in Figure 9 and Figure 10 for incremental and decremental forces respectively.

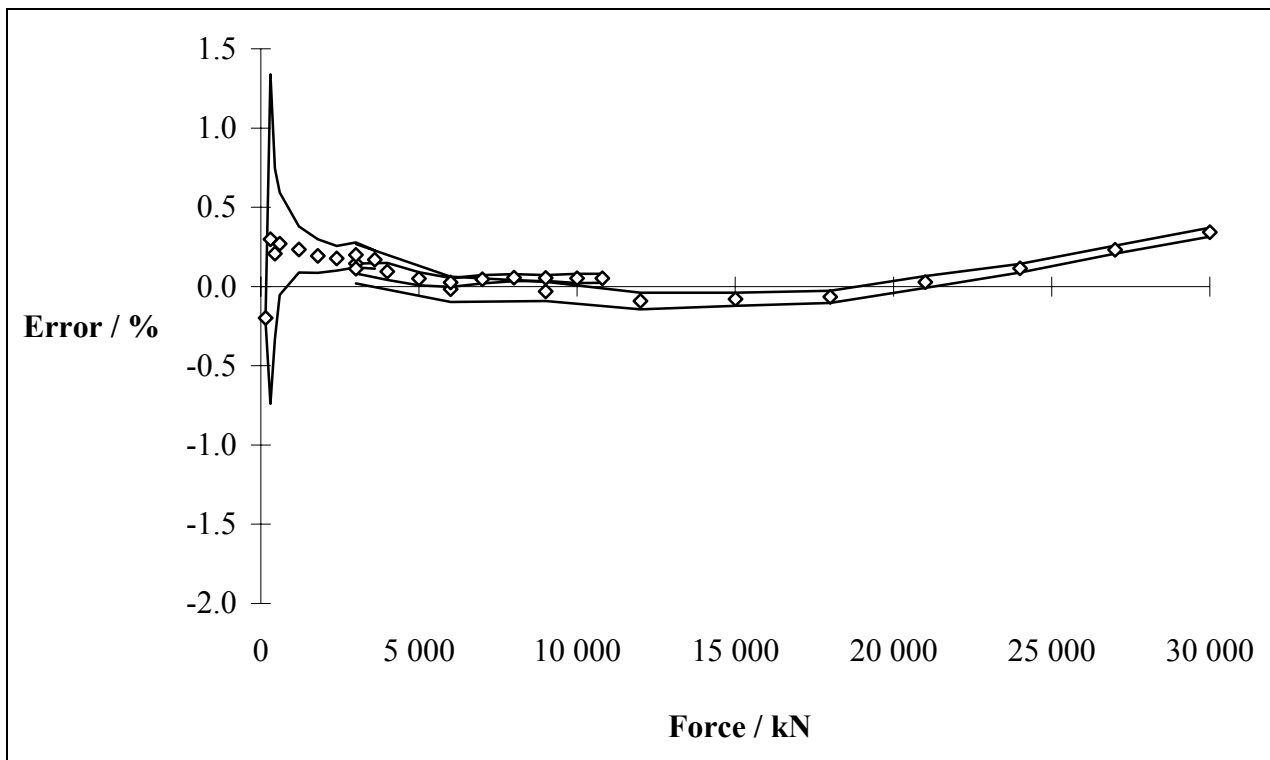


Figure 9: Incremental errors of 30 MN machine forces, with 95 % confidence intervals

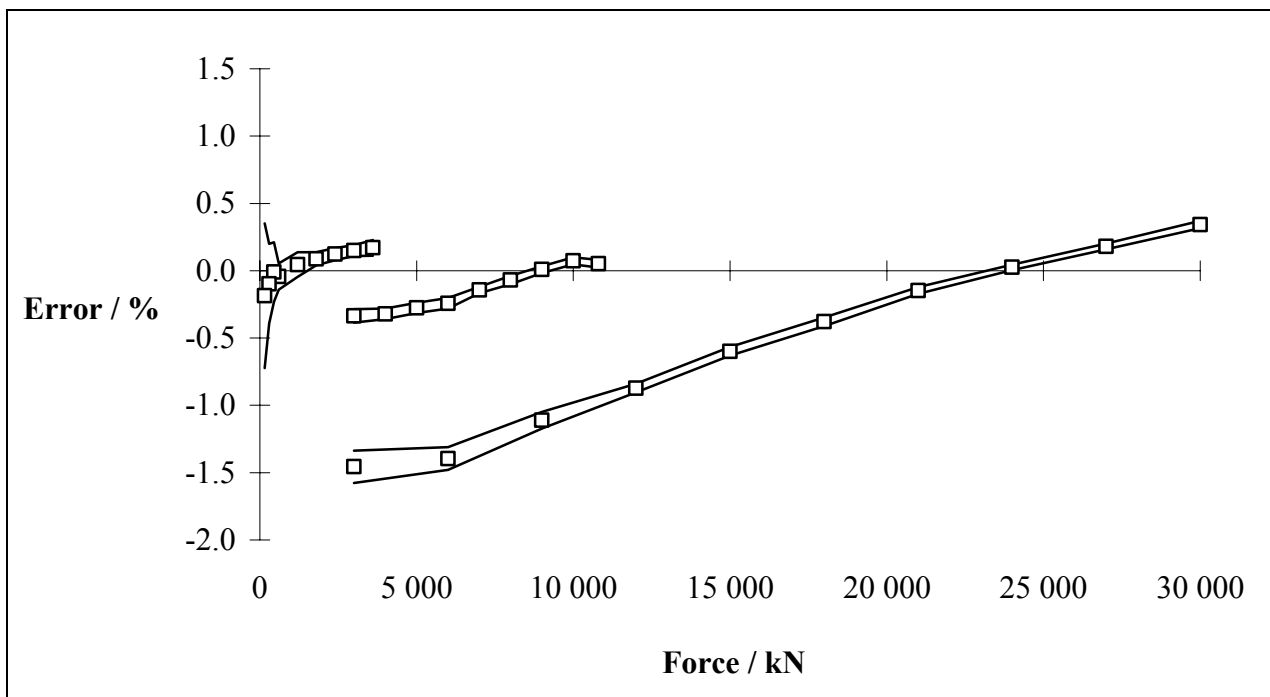


Figure 10: Decremental errors of 30 MN machine forces, with 95 % confidence intervals

Figure 11 and Figure 12 illustrate the importance of applying corrections to this machine. While there is an overall improvement to the incremental curve the decremental curve changes dramatically. A hysteresis of -1.5 % has been corrected to give a near zero error under the same loading conditions.

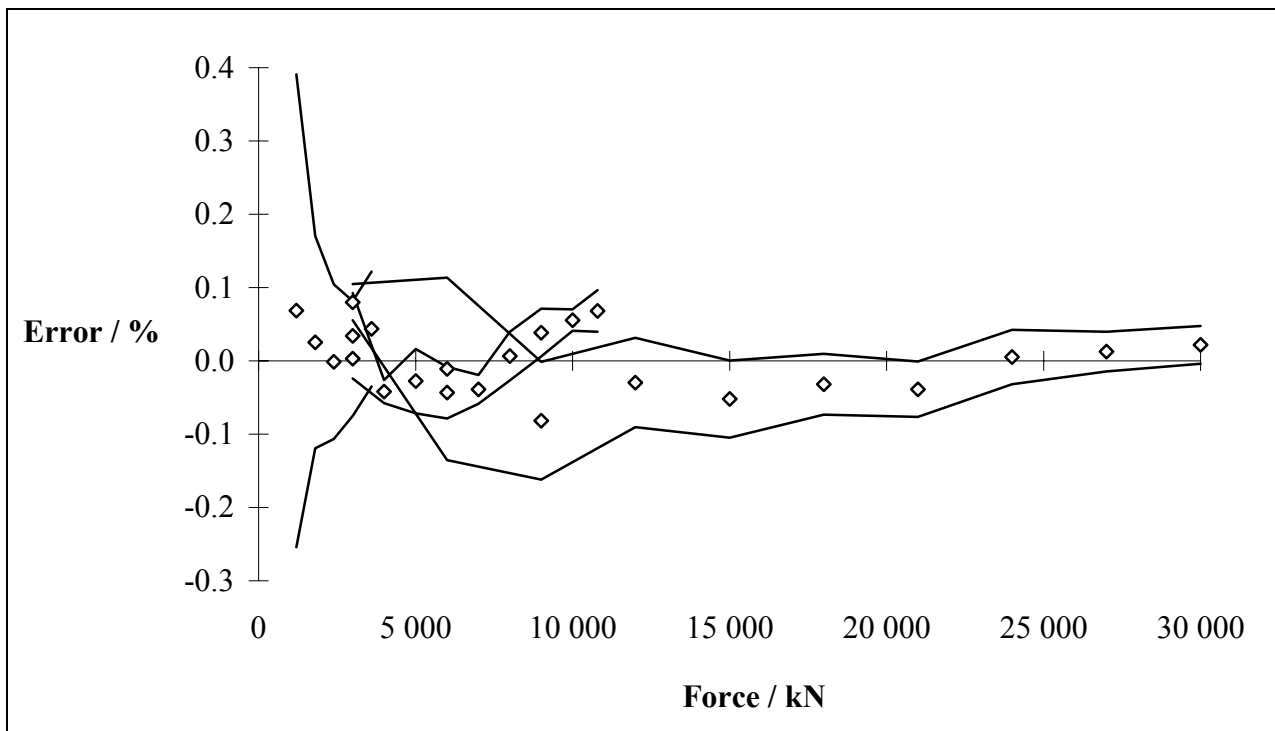


Figure 11: Post-correction incremental errors of 30 MN machine forces, with 95 % confidence intervals

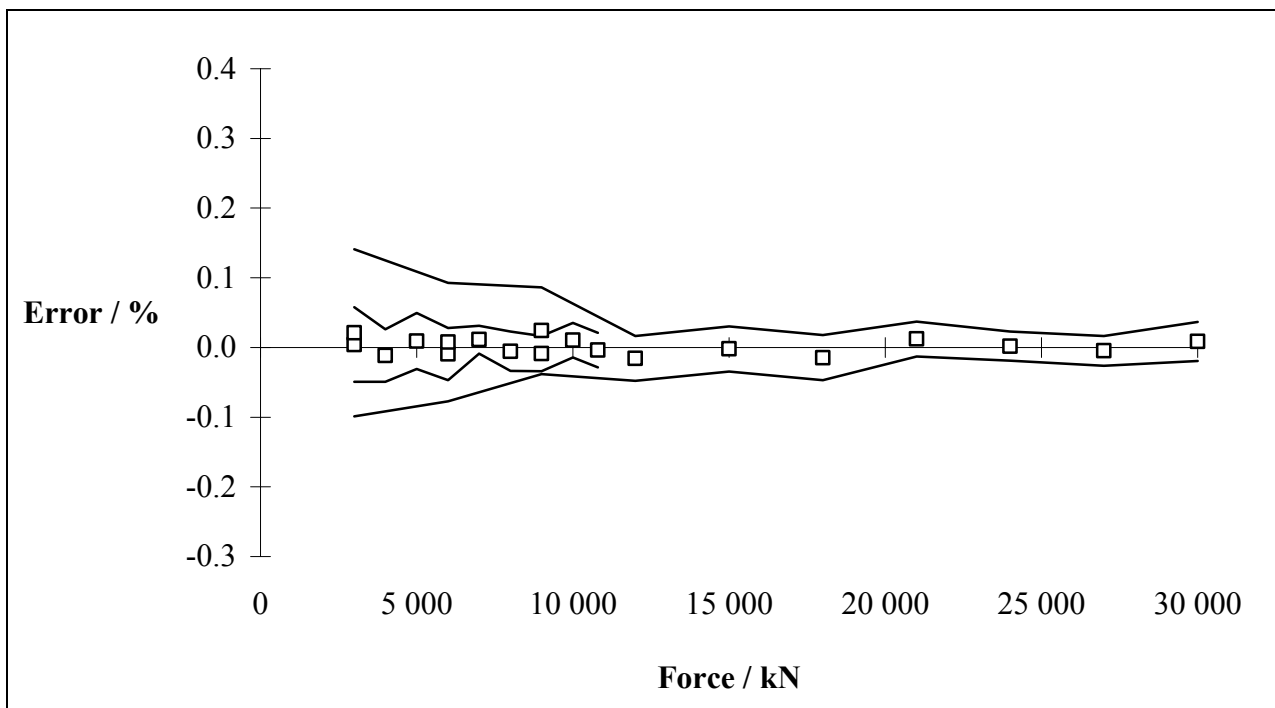


Figure 12: Post-correction decremental errors of 30 MN machine forces, with 95 % confidence intervals

4. Discussion

4.1 5 MN hydraulic amplification machine

From Figure 4 and Figure 5, it can be seen that the 5 MN force standard machine generates forces with an uncertainty of less than 0.05 % over a range from 150 kN up to 5 MN once corrections have been applied.

At forces below 150 kN, the calculated uncertainties are significantly greater than 0.05 %. This is partly due to the calibration equipment, which does not perform particularly well at the initial calibration points, and partly due to the machine's control system - a control stability of a constant pressure value is more significant in percentage terms at lower pressures.

The target uncertainty for this machine was 0.02 %. This was not achieved during the 'build-up' exercise reported upon, as a result of the reproducibility contributions and the differences between the tie-up points. However, initial results from a repeat exercise indicate a better performance from the machine - this, allied with work into modifying the machine, could enable the tighter specification to be met in the near future.

4.2 12 MN machine

The repeatability and reproducibility of the 12 MN machine were found to be very good in the short term. The long term stability was less encouraging, and further work over an extended period is being carried out to determine the underlying performance of the machine. Although the machine exhibits a high degree of hysteresis, dependent upon the maximum applied force, it is possible to correct for this.

The target uncertainty for this machine was 0.05 %. This was not achieved during the 'build-up' exercise reported upon, as a result of the machine's long term stability and the uncertainty of the 5 MN machine. A future improvement in the performance of the 5 MN machine should enable the uncertainties associated with this machine to be reduced.

4.3 30 MN machine

Figure 9 and Figure 10 show the error and uncertainty bounds at each force measured during the calibrations. It can be seen clearly that the incremental forces were found to coincide to a far greater degree than the decremental forces. This is due to the inherent hysteresis of a strain gauged hydraulic machine. Hysteresis is a fundamental property of all strain gauged devices and the relative hysteresis at a given force is dependent upon the maximum force applied in the test. A simplified model of the hysteresis of the 30 MN machine is that the hysteresis is equal to -0.05 % of nominal force per meganewton below the maximum force in a test. This must be determined as accurately as possible when calculating corrections for use in calibration work and, moreover, corrections must be re-calculated for each individual maximum force used.

The outcome of this is that the 30 MN machine is a difficult one to operate to its full potential. The incremental forces can be corrected to minimise their errors over the full range of the machine, but the hysteresis present leads to the need to have what is effectively a database of decremental corrections, each dependent upon the maximum applied force.

The target uncertainty for this machine was 0.1 %. This was not achieved incrementally during the 'build-up' exercise reported upon, as a result of the reproducibility contributions and the differences between the tie-up points, but it was achieved decrementally from 30 MN down to 4.5 MN. A future

improvement in the performance of the 5 MN machine should enable the uncertainties associated with this machine to be reduced.

5. Monitoring exercises

A fundamental part of the maintenance of force standards is to ensure that all machines continue to perform at their certified level of uncertainty between full re-calibration exercises. This is achieved by assigning a specific load cell to each machine for the sole purpose of monitoring its performance. Regular monitoring at specified intervals should highlight any deficiencies or changes in the machine's performance.

For example, such a monitoring exercise indicated poor performance in the 5 MN machine, leading to it being withdrawn from service. Investigation of the problem revealed a large, intermittent drift in applied force which was remedied by changing the positions of the deadweights. As a result, the machine performed with lower uncertainties in the applied force than it had at the previous calibration, and it was therefore brought back into service.

6. Conclusions

The 5 MN machine was certified as having an uncertainty in the generated force better than 0.05 % at a 95 % confidence limit, when corrections are applied. The calculated uncertainty values were found to be lower than this over most of the range with the lower force values having the highest uncertainty in percentage terms.

The 12 MN machine exhibited incremental uncertainties of less than 0.2 % and decremental error characteristics to which smooth curves could be fitted.

The 30 MN machine was certified as having an uncertainty in generated force better than the greater of 0.2 % and 2.5 kN incrementally and better than the greater of 0.1 % and 4.5 kN decrementally. These values apply for a test with a 30 MN maximum force value with corrections applied. The decremental corrections and the associated uncertainty of generated force change according to the maximum force applied in the test.

Work is presently being undertaken to modify and improve the 5 MN machine. This should not only decrease the uncertainty associated with the forces generated by this machine, but also have a beneficial effect on the claimed uncertainties of the forces generated by the 12 MN and 30 MN machines.