NPL Report CLM 6

May 1998

A Large Reference Artefact for CMM Verification

N R Cross, J R Dotson, D R Flack and G N Peggs
Centre for Length Metrology

M G Cox and A B Forbes
Centre for Information Systems Engineering

National Physical Laboratory
Queens Road
Teddington
Middlesex
United Kingdom
TW11 0LW

R Corta
Tekniker, Eibar, Spain

J O'Donnell
National Metrology Laboratory, Dublin, Ireland and

E Prieto
Centro Español de Metrologia, Madrid, Spain

Abstract. This report describes the design, construction and calibration of a large reference artefact suitable for use as a length artefact in the verification of co-ordinate measuring machines (CMMs). The artefact is of modular design, constructed in 1 m and 0.5 m tubular sections made from carbon-composite materials. Each section can carry a range of reference surfaces including reference spheres (tooling balls). The artefact has been calibrated by three metrology laboratories using a repositioning methodology. Equipment for supporting the artefact during the verification of a CMM has also been designed and constructed. The artefact and positioning equipment have been successfully tested in the verification of two large CMMs according to the principles of ISO 10360-2. The design philosophy and calibration methodology are flexible and can be easily adapted, providing a practical approach to large CMM verification.

ISSN 1369-6777

National Physical Laboratory
Teddington, Middlesex, United Kingdom, TW11 0LW

Approved on behalf of the Managing Director, NPL
by Dr K H Berry, Head, Centre for Length Metrology

CONTENTS

ANNEXES

TABLES

FIGURES

(This page is intentionally blank.)

1 INTRODUCTION

A major step in establishing the traceability of measurements carried out by co-ordinate measuring machines (CMMs) involves the comparison of CMM measurements of an artefact with the calibration information associated with the artefact (Cox, Forbes and Peggs, 1997, Cox, Forbes, Harris and Peggs, 1998). This approach is explicit in ISO 10360-2 (ISO, 1995) which gives a procedure for determining the length measuring capability of a CMM from measurements of calibrated length artefacts such as gauge blocks and step gauges. Although the standard applies most directly to CMMs with operating axes of 1 metre or less, its underlying principles apply to co-ordinate measuring systems with working volumes of any size. To carry out a verification according to the standard requires (amongst other things):

- a reference artefact whose length is no less than 66% of the longest space diagonal within the working volume,
- that the artefact has a calibration uncertainty no greater than 20% of the error of indication of the CMM,
- measurements of the length of the artefact along seven distinct measuring lines.

The method is limited only in our ability to provide an artefact that can be accurately calibrated and stably positioned within the working volume. While step gauges and gauges blocks of up to 1 metre, suitably calibrated, are not uncommon, the construction of longer artefacts with comparable accuracy and practicality presents significant difficulties. This report describes:

- the design and construction of a large reference artefact (LRA) up to five metres long. The artefact is structurally stable, easily supported and positioned and is of modular construction for flexibility and ease of portability,
- the design and construction of equipment to position the artefact within the working volume of a large CMM,
- a method for the accurate calibration of the artefact using repositioning techniques (Cox, Cross *et al.*, 1997), and
- its use in the verification of large CMMs according to the principles of ISO 10360-2.

In Section 2 we describe the design and construction of the large reference artefact and in Section 3 we report on the artefact calibration. Section 4 reports on the verification of large CMMs using the artefact, while Section 5 considers aspects of the artefact design and performance. Our concluding remarks are presented in Section 6.

2 ARTEFACT DESIGN AND CONSTRUCTION

2.1 CARBON COMPOSITE MODULAR ARTEFACT

An early design decision was to build the artefact of carbon-composite materials, thus providing rigidity and at the same time keeping the artefact lightweight. To meet the modularity requirement, the artefact is constructed from tubular sections; see Figure 1. During the construction of the tube, the carbon fibres are layered uni-directionally and aligned with the tube axis. An additional layer of fibre material is wrapped around the tube at 90° to the tube axis which helps to control the ovality of each individual completed tube. The tubes used for the LRA are 1 m (four have been made) and 0.5 m (two have been made) in length. This range of sizes will allow for the verification of the performance of machines with largest dimensions up to 5 m in 0.5 m steps. It is possible, in principle, to add further sections.

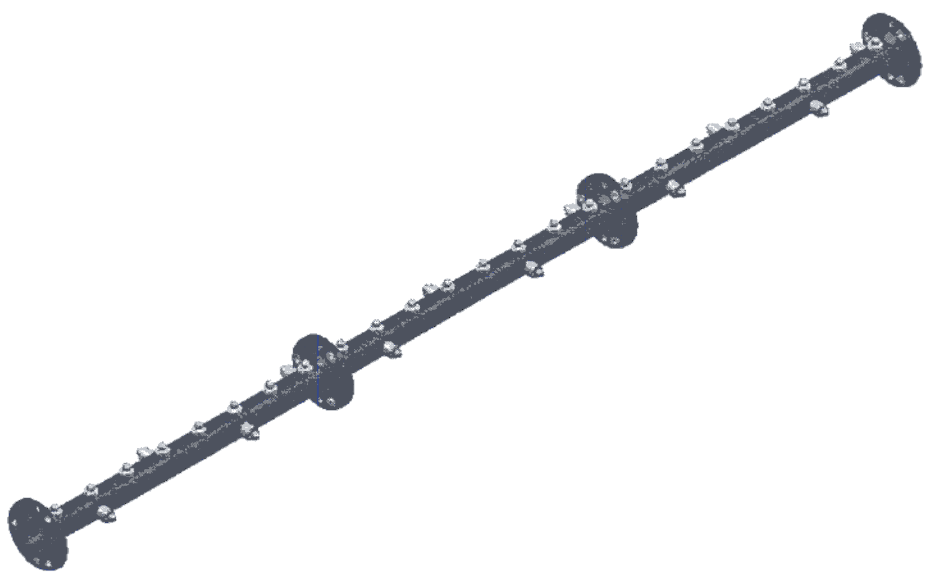


Figure 1: Three modules of the large reference artefact joined together in a linear assembly. Each module is shown carrying 8 reference spheres fixed to the measuring stations on top of each cylindrical section together with four spheres attached to the sides.

The requirement for modularity introduces the problem of how the modules are to be joined together. For this purpose, each tube has a flange of laminated construction bonded to each end. The flanges are manufactured from sheets of carbon fibre, each laid at a different orientation and pressed over a former, the required thickness being built up in layers. The alignment and connection of the modules is achieved by semi-kinematic flange joints. Pure kinematic connections were ruled out as it was thought that they would not provide the required rigidity and would be subject to damage during use. The operation of the flanges is as follows.

The mating faces of the flanges are precision ground so that they are flat and also square to the tube axis, so that when bolted together the modules have no freedom to move axially. At one end of a module the flange has two precision ground holes located on a diameter. The upper hole is fitted with one half of a conical bush and the lower hole with a tooling pin which protrudes approximately 12 mm. The mating flange on the adjoining module has two corresponding holes. The upper hole is fitted with the other half of the conical bush and the lower hole with a precision-ground, parallel bush which only just accepts the protruding tooling pin (with approximately 0.005 mm clearance). The tooling pin is manufactured so that it only locates across the equatorial plane of the bush thus allowing only a slight movement in the vertical direction.

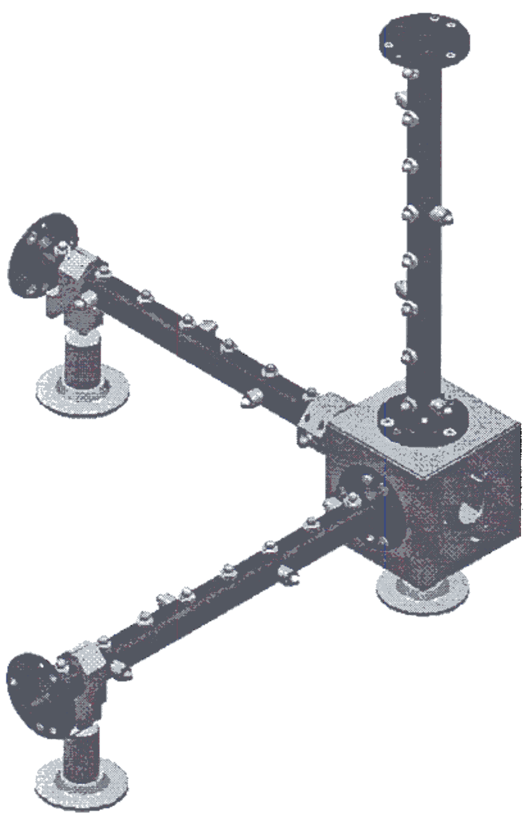


Figure 2. Titanium cubical mount with three 1 m modules attached in a 3-dimensional configuration.

During assembly the two mating flanges are brought together so that the tooling pin locates in the parallel bush. The two halves of the conical bush are then connected by means of a conical bolt which has been lapped to be an exact fit with the flanges in contact. Finally, the flanges are pushed together (using a torque wrench set to 12 Nm) by three equi-spaced M10 bolts. The bolts play no part in the location of the flanges but merely hold the modules together. The cone/bolt assembly fixes the vertical degree of freedom of the flanges and the tooling pin/parallel bush prevents any significant rotation.

A cubical 'base-station' has been designed and manufactured from titanium to which the tubular sections can be attached. Using this base-station, a reference artefact can be assembled as an L-, T- or X-shaped 2-dimensional artefact or a 3-dimensional artefact of various configurations; see Figure 2. The method of mounting the modules to the cube is similar to that for the flange-to-flange joins.

There are eight measuring stations mounted axially 125 mm apart on each 1 m module and four on each 0.5 m module, in line with a generator of the cylinder. The measuring stations are positioned symmetrically such that the 125 mm spacing is preserved across the joins. These stations consist of titanium platforms bonded to the tube and are designed to carry a range of reference surfaces such as steps (gauge blocks), cylindrical dowels or spheres.

The complete assembled artefact can hold 40 reference surfaces that can be used to define calibrated lengths associated with the artefact.

2.2 REFERENCE SURFACES

The prime function of the artefact is to provide a stable structure to carry reference features which, in turn, comprise the reference surfaces. A number of design options were considered:

Steps. The step gauge is an important artefact in verifying CMMs with moderate working volumes. Thus, an obvious choice for the reference surfaces on the LRA are steps with two or more flat surfaces, for example gauge blocks. The corresponding calibration information is a set of distances between selected plane, nominally parallel faces. The main advantages of this choice are:

- It represents a simple extension of step gauge principles familiar to users.

- It allows for simple, bi-directional probing strategies, a requirement of the current version of ISO 10360-2.

The main disadvantages are:

- The steps have to be mounted accurately to achieve parallelism between faces.

  The calibrated distances represent only 1-dimensional information.

- The steps cannot be used easily to define calibration information if the artefact is configured as a 2- or 3-dimensional artefact using the base station.

  Any bending of the artefact will affect the parallelism of the faces (as well as the distance between the faces).

Spheres. Reference spheres (tooling balls) with high sphericity are readily available and have long been used in CMM verification as the reference surfaces for ball plates. The main advantages of this choice are:

- No special mounting alignment is required. The only requirement is that the location of the spheres is known accurately enough for CMM CNC programming.

  The calibration information can be in terms of the inter-ball distances as (3-dimensional) positions in some fixed frame of reference. This applies equally to 1-, 2- or 3-dimensional configurations of the artefact.

  Spheres can be probed in a range of orientations.

The main disadvantages are:

- The calibration information depends on calculations of the sphere centre co-ordinates; however reliable algorithms and software are available for this task (see, e.g., Forbes, 1989).

- The measurements are not truly bi-directional. This is of less importance for large volume CMMs where probing errors are likely to be small compared to the geometrical errors.

For the artefact described here, each measuring station carries a grade 25 (±0.001 mm sphericity) silicon nitride sphere of 20 mm diameter. In addition to the 8 reference features on each 1 m module, there are four auxiliary spheres or, in the case of the 0.5 m module, two auxiliary spheres. These auxiliary spheres are of the same design as the main spheres but lie one in each of the planes that pass through the centre of every other of the main spheres and are radial to the tube. With respect to the line of the measuring stations, these auxiliary spheres are mounted alternately at +90° and -90°; see Figure 1. A schematic diagram of a 1 m module is given in Figure 3 and the nominal positions of the spheres are given in Table 1. These auxiliary balls are an essential feature of the modules because they enable a calibration of the artefact to be performed on a standard-sized CMM and allow the measurements of the individual modules to be combined. They can also be regarded as reference features used to define additional calibration information.

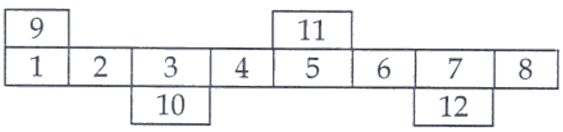


Figure 3. Location and numbering of the reference spheres on a 1 m module.

The use of carbon composites, titanium and ceramic materials throughout has ensured that the artefact is of modest weight: the complete assembled 5 m artefact including reference features weighs less than 25 kg.

2.3 ARTEFACT POSITIONING

Another major factor in the design of the artefact system is its mounting. The main features of the design (see Annex 1) are:

- The artefact is mounted on a substructure carried by one central, light, stable pillar on a triangular base. The base has three retractable wheels that allows the artefact to move easily on the CMM table. The pillar can either be attached rigidly to the CMM table or weighed down using lead masses. The column allows the artefact to be lowered, raised and orientated at any angle.

  The main supporting substructure can be assembled using two or three aluminium sections depending the length of the artefact to be mounted.

  The artefact is mounted on the substructure using clamps that attach to the flanges at each end of the artefact. The artefact can also be suspended vertically using the same equipment.

  The design is capable of orientating the artefact in all the positions suitable for

CMM verification according to ISO 10360-2 procedures.

Sphere	x	y	z
1	0	0	30
2	125	0	30
3	250	0	30
4	375	0	30
5	500	0	30
6	625	0	30
7	750	0	30
8	875	0	30
9	0	75	-25
10	250	-75	-25
11	500	75	-25
12	750	-75	-25

Table 1. Nominal position (in mm) of the reference spheres of a 1 m module in standard position.

The stability of the support has been tested using a gauge attached to one end of the support to measure any change in position. No change was detected (at the micrometre level) over a duration of many hours.

2.4 ARTEFACT ALIGNMENT

Since the tooling balls are situated off the neutral axis of the tube and the tube bends due to gravitational loading, the inter-sphere spacing will vary with angle, support condition and length. Experiments and finite element analysis have shown that bending only introduces significant changes in inter-sphere spacing for lengths greater than 2 metres. In order to minimise these effects, a mechanical-optical alignment procedure has been implemented to determine the 'straight' condition for any arbitrary length of artefact and then support the artefact in such a manner that the centre sag is within 0.1 mm of the 'straight' condition. This is sufficient to allow for an accurate correction of the inter-sphere distances using finite element analysis.

2.4.1 *Equipment required*

In addition to the modules of the large artefact required to give the necessary length, and the hardware required to fasten these modules together, the following hardware is required:-

- Taylor Hobson alignment telescope (serial number 112/537-213 supplied by NPL, complete with straight and right-angle eye piece adapters).

- Alignment telescope to artefact flange adapter (supplied by NPL).

  Artefact upper support stub complete with cross-hair (supplied by NPL).

- Two titanium adjustable angle tube mounts (supplied by NPL) or other suitable method of mounting to the aluminium stub tubes.
- Centre adjustable aluminium angle mount (supplied by NPL) or other suitable method of pushing the centre of the tube without distorting the.
- Self centring cross-hair target (supplied by NPL).
- Micrometer screw jack.

2.4.2 *Assembly of artefact*

When the required length of artefact has been chosen, the relevant modules of artefact are prepared such that they can be assembled in the correct sequence using the supplied bolts, cones etc. The self centring target is first carefully slid into the centre module and positioned within 100 mm of the calculated centre of the completed length. For artefact lengths which would have a flange at the centre, the target is positioned so that it is as close as practicable to the flange of one of the two modules. The target is then rotated within the tube until one arm of the cross-hair points radially out towards the tooling balls. The modules of the artefact are then assembled in the conventional manner.

The upper support stub is then joined to one end of the artefact in such an orientation that the cross-hairs are parallel to those of the centre target and the telescope adapter is assembled to the other. The alignment telescope is then inserted into its adapter and rotated such that the graticule is parallel with the cross-hairs. The telescope is held in place by tightening the split clamp round the aluminium tube.

2.4.3 *Defining the 'straight' condition*

The alignment of the artefact in the absence of gravitational distortion is defined as 'straight'. Practically this can be realised if the artefact is aligned vertically. With the artefact vertical (either hung from the upper end, or resting on the lower) and with the telescope either at the top or the bottom, the offset of the two cross-hairs is measured with the telescope micrometers and noted, with prime importance being given to the deviation in the direction of the tooling balls (the telescope is focused on each of the two cross-hairs in turn and the two micrometer knobs adjusted to centre the telescope cross-hairs within the target cross-hairs).

Note: if disassembly of the artefact is necessary between the vertical set-up and that used on the CMM, then only the inter module flanges should be disturbed. The upper support stub to tube or telescope adapter to tube joints must not be disturbed.

2.4.4 *Mounting the artefact on the CMM*

Once the artefact is mounted on the positioning equipment, the aluminium adjustable angle mount is then used to support the centre of the tube from a micrometer type screw jack with better than 0.05 mm vertical resolution and greater than 10 kg load carrying capacity.

2.4.5 Aligning the artefact on the CMM

With the artefact supported at the required position on the CMM, the positions of the two cross-hairs are measured again with the alignment telescope. Because of the sag of the artefact, the centre cross-hair will be 'lower' than before. The centre of the artefact is jacked up (using a micrometer screw jack attached to the adjustable angle support) until the centre cross-hair is at the same offset relative to the far cross-hair as in the vertical condition. Because of the bend in the artefact, the absolute positions of the cross-hairs relative to the telescope will have changed and no attempt should be made to reproduce exactly the cross-hair positions originally measured. The requirement for alignment is that the relative position of the two cross-hairs be maintained. For instance if the original measurements were +0.4 mm (far) and -0.2 mm (centre) - a difference of 0.6 mm, then example final readings of -0.1 mm (far) and -0.7 mm (centre) would be acceptable.

For artefact lengths of four metres or greater, the centre cross-hair should be raised 0.1 mm above the 'straight' position in order to counteract the sag in the two 2 metre halves. Calculations have shown that if the above procedure is followed, the absolute maximum ball deviation from the calibrated position is 15 µm. Finite element calculations can be used to correct for these deviations.

3 ARTEFACT CALIBRATION

The large reference artefact has been calibrated by NPL, NML and CEM/Tekniker.

3.1 GENERAL CALIBRATION PROCEDURE

This section describes the general calibration procedure involving the four 1 m modules (modules 1 to 4); the procedures involving the 0.5 m modules (modules 0 and 5) are essentially the same.

The measurements involve firstly the measurements of the individual modules and then the measurements of the three joins 12, 23 and 34.

During the measurements of the 1 m modules, the module is supported by vee-blocks under spheres 2 and 7 (see Figure 3.) During the measurements of the joins, the modules are supported under spheres 4 and 17 (that is, sphere 5 of the second module); see Figure 4. Finite element calculations have been performed to determine the deflection for these support arrangements; see Annex 2

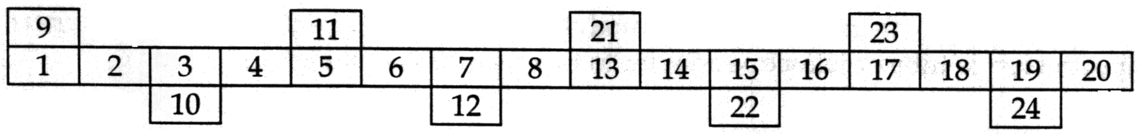


Figure 4. Location and numbering of spheres of two 1 m modules joined together.

Once the measurements have been made, the data is analysed by the position calibration software (Butler *et al.*, 1998) and estimates of the locations of the reference spheres (registration points) determined. From this information, the measurement data are then transformed into a standard position and the finite element corrections applied to them to yield a correct set of data. The position calibration software is then applied to the corrected data to estimate the locations of the reference spheres for the modules in a zero-deflection state. In this way, the deflections that occur during the calibration measurements are taken into account.

3.2 EXPERIMENT INFORMATION

The modules were measured by NPL, NML and CEM, the last through CEM's collaborator, Tekniker. The measurements at NPL were performed during July and August, 1997 (immediately after the construction of the modules) using a Zeiss CMM UPMC 550, with a longest axis of 550 mm. The modules were then shipped by road and sea to NML, Dublin. The measurements at NML were carried out using a Zeiss UMC 850 with a longest axis of 1200 mm, during August and September, 1997. The modules were then sent to Tekniker by road and air freight for measurements during October and November, 1997. The CMM involved was a Zeiss UPMC - CARAT with a longest axis of 1200 mm.

3.3 RESULTS FOR 1 METRE MODULES

The measurement results for the four 1 m modules from the three laboratories are presented in Annex 3. All results have been corrected for deflection during calibration. Firstly, the estimates of the locations of the 12 reference sphere centres for each module in standard position are presented in Annexes 3.1—3.3. The results are also presented in terms of the differences of selected inter-sphere distances in Annex 3.4.

The results show that for modules 1, 2 and 3 the estimates of the distances derived from the NML and CEM results agreed to 0.0012 mm or better, well within the errors that can be accounted for by accuracies of the CMMs involved. The NPL results differ from NML and CEM by up to 10 micrometres and the results are consistent with a linear change of scale. For module 4, there is close agreement between NPL and CEM (1.3 micrometres or better) but the NML results differ by up to 0.0037 mm. Potential causes for the discrepancies, including, CMM accuracy (scale error), differences in deflection due to different mounting, environmental effects, mainly temperature and humidity and long term artefact stability were identified and investigated. In order to determine better the likely cause, module 1 was shipped back to NPL by road and air freight and re-measured by NPL in November, 1997, using the same CNC programme and set up procedures as for the earlier measurements. The comparison of the second set of NPL measurements with the NML and CEM measurements is also given in Annex 3.4.5. These results show that on the basis of the NPL measurements alone, the module had increased in length by approximately 0.007 mm, so that the second set of NPL results are in much closer agreement with the others.

The most likely explanation for the change in dimension is due to (a) the artefact stabilising after machining and assembly (the NPL measurements were taken shortly after the reference spheres and flanges were glued to the module) and (b) the absorption of moisture by the materials. Further long term experiments will be required to characterise and quantify these medium to long term effects.

3.4 RESULTS FOR THE MODULE JOINS

The measurements of the module joins are presented in Annex 4, including comparisons in terms of the differences of the estimates of selected inter-sphere distances. The results show that variations up to 0.007 mm have occurred. The difference between the NPL results and the other results is due partly to the change in module dimensions apparent in the measurements of the modules. For example, the distances between spheres 7 and 8 and spheres 13 and 14 do not involve the join but differ by up to 0.0016 mm. On the other hand the NML and CEM results are reasonably consistent for these distances but all distances that straddle a join differ by roughly the same amount (up to 0.003 mm).

While the results within each laboratory are self-consistent (discussed fully in section 3.5) there are significant differences between the laboratories. The main reasons for this can be attributed to variations in the laboratory procedures and operator effects, relating to the method of assembly of the joins, the amount of settling time and methods of support. For this reason it is recommended that, where possible, the same operator assembles the artefact for calibration and use.

3.5 REPEATABILITY OF THE JOINS

The repeatability of the joins has been tested at all three laboratories. The experiments involved joining two modules, measuring the join area and then disassembling the join. This process was repeated between 2 and 6 times and the results analysed using the position calibration software. The repeatability of the joins can be measured by the standard deviation of the residual errors of the fit to the data and also by the standard uncertainty of the distance between the two spheres of maximum separation (spheres 7 & 14 for the NPL measurements, spheres 5 & 16 for NML and CEM). These quantities are given in Table 2. The results show that the inter-sphere distances generally have a standard deviation of 0.0005 mm or better. Only the NML results for join 3 & 4 are significantly different, due most likely to the poor operation of the seating mechanism for the join. Even so, the standard uncertainty associated with the inter-sphere distance is modest. If we look instead at the inter-sphere distances determined directly from the measurements and then calculate their standard deviation, we find that the standard deviation associated with distance between sphere 5 of module 3 and sphere 4 of module 4 for the NML measurements is only 0.0001 mm, showing that the join is repeatably defining the inter-sphere lengths.

		J12 (μm)	J23 (μm)	J34 (μm)
NPL	*s*	1.88	0.66	0.86
	u	0.53	0.16	0.24
NML	*s*	1.0	0.50	3.44
	u	0.45	0.18	1.5
CEM	*s*	0.06	0.09	0.40
	u	0.04	0.52	0.33

Table 2. Measurement of the repeatability of the joins. The table gives the standard deviation (*s*) of the residual errors and the standard uncertainty (*u*) of the distance between spheres 7 & 14 (NPL results) or between spheres 5 and 16 (NML and CEM results) for each of the three laboratories.

We also present a more detailed analysis of the measurements of join 2 & 3 by CEM to gain a better understanding of the behaviour of the joins. Join 2 & 3 was measured six times by CEM. We have used the position calibration software to combine each of these six sets of measurements with the measurements of the individual modules to obtain six estimates of the geometry of the 2 m assembly. In particular, for each pair of reference spheres on the assembly we can calculate six estimates of their separation. The standard deviation of these six distances is a measure of the repeatability of the join. Annex 4.5.1 gives the standard deviations firstly for all six measurements. We see that the longest distance between sphere 1 and 20 (nominally 1.875 m) has a standard deviation of 0.00079 mm. This figure compares with that of 0.00052 mm in Table 2. The reason for a difference is that the calculation presented in Table 2 properly takes into account correlation in the parameters describing the sphere locations while the other does not.

A more detailed analysis of the data shows that the third set of measurements is atypical; omitting this set, the standard deviation is 0.00041 mm (Annex 0. One reason why the third set of results agreed less well with the others is that only 30 minutes settling time elapsed after the assembly of the join before measurements were taken; see Table 3.

Measurement number	1	2	3	4	5	6
Stabilisation time (hrs)	3.0	3.0	0.5	1.5	12.0	12.0

Table 3. Stabilisation times for the six measurements of the join of modules 2 & 3.

Pair		x-, y- and z- co-ordinates (mm)
1 & 2	max |Δ|	0.0003 0.0048 0.0026
	s	0.0002 0.0022 0.0012
2 & 3	max |Δ|	0.0012 0.0027 0.0034
	s	0.0010 0.0014 0.0015
3 & 4	max |Δ|	0.0006 0.0047 0.0012
	s	0.0006 0.0024 0.0006
4 & 5	max |Δ|	0.0006 0.0050 0.0033
	s	0.0005 0.0022 0.0014
5 & 6	max |Δ|	0.0002 0.0011 0.0025
	s	0.0001 0.0005 0.0011
6 & 1	max |Δ|	0.0002 0.0043 0.0010
	s	0.0001 0.0023 0.0005

Table 4. Comparison of six sets of estimated reference sphere positions of a 2 m assemblies determined by the position calibration software from measurements of modules 2 & 3 and six measurements of their join. The numerical columns give the maximum absolute difference (max |Δ|) and standard deviation (*s*) of the differences of the *x*-, *y*- and *z*-co-ordinates for the pairs of assemblies indicated in the first column.

We can also look not only at the distances between sphere centre estimates but also the differences in the sphere centres estimates themselves. Table 4 gives the maximum absolute difference and the standard deviation of the differences between the x-, y- and z-co-ordinates of the centres of the sixteen spheres lying on top of the modules for pairs of estimates of the 2 m assembly. The table shows that the differences in x-co-ordinates are generally smaller than 0.0005 mm (consistent with the small variation in inter-sphere distances), while those for the y- and z-co-ordinates are better than 0.005 mm. Recall that the joins are semi-kinematic with a cone arrangement giving a precise match at one connection and a pin arrangement diametrically opposite constraining rotation about the cone axis. This pin has a few microns play, enough to account for the observed alignment errors in the y- and z-co-ordinates.

The analysis of this join shows that the repeatability of the joins is sufficient to determine distance calibration to sub-micrometre accuracy and three dimensional positions to an accuracy of a few micrometres.

3.6 CALIBRATION OF 4 METRE ASSEMBLY

The position calibration software has been used to determine estimates of the sphere centres for the assembly of all four 1 m modules (M1, M2, M3 and M4). The results, in terms of selected inter-sphere distances and their standard uncertainties are given in Annex 5. These results take into account the repeatability of the joins, as all measurements of the joins are processed. Summary results are given in Table 5. They show that the NML and CEM estimates of the distance between the first and last sphere, nominally 3875 mm, agree to better than 0.01mm, while the NPL result differs from the others by about 0.04 mm. The difference in the NML and CEM results is accounted for mainly in the assembly of the joins, pointing again to an operator effect. The difference in the NPL results is due mainly to the long-term change in module length after manufacture.

The table also shows the standard deviation s for the residual errors, and the estimated standard uncertainty *u* of the computed distances. Their modest size confirm that the repositioning methodology and the operation of the joins contribute little to the uncertainty budget.

M1,2,3,4	NPL	NML	CEM
s (μm)	0.99	1.24	0.49
d (mm)	3875.1455	3875.1818	3875.1911
u (μm)	1.47	0.81	0.56

Table 5. Estimates of the longest distance (*d*) defined by the 4 m assembly, along with their standard uncertainties (*u*). The standard deviations (*s*) of the residual errors are also given.

3.7 ARTEFACT UNCERTAINTY BUDGET

A key requirement of ISO 10360-2 is that the length artefact used in the CMM verification has calibrated lengths and it is therefore important to estimate the uncertainties associated with these calibrated lengths. The main contributions to the uncertainty budget for the large reference artefact are now discussed.

3.7.1 CMM measurement uncertainty

The methodology for calibrating the reference artefact involves the use of a CMM to measure the modules and joins. Therefore measurement error associated with the CMM measurements will contribute to the uncertainty budget. This contribution can be considerably reduced if the module measurements are compared with a length artefact such as a gauge block or a step gauge using the comparator technique. In this situation the contribution to the uncertainty budget is related to the uncertainty associated with the length artefact, generally significantly less than that associated with the CMM.

3.7.2 Uncertainty in temperature measurement during calibration

The temperature during calibration needs to be monitored. Assuming that the temperature is known to 0.1° C or better, then this contribution is small.

3.7.3 Uncertainty in expansion coefficient

The coefficient of linear thermal expansion, being a measured quantity, has an uncertainty that also contributes a small amount to the total uncertainty.

3.7.4 *Deflection during calibration*

The deflection during calibration due to gravitational loading is corrected for using calculations determined by a finite element model validated by experiments. Errors in the modelling will make a small contribution.

3.7.5 *Repeatability of the joins*

The artefact will need to be reassembled after calibration before its use in CMM verification. In order for the calibration to be valid, it is necessary that the assembly takes up the same geometry as it did during calibration. For a single operator assembling the joins according to a fixed procedure, the contribution to the uncertainty arising from the repeatability of the joins is modest.

3.7.6 *Uncertainty in temperature measurement during use*

For accurate results, the temperature of the artefact will need to be monitored during its use. Inaccuracies in these measurements will contribute to the error budget.

3.7.7 *Deflection during use*

Longer assemblies will deflect significantly (up to 3 mm) if supported only at two positions. In order to minimise this effect, the mechanical-optical alignment procedure can be used to provide one or more extra supports and align the artefact much closer to a zero deflection state. A finite element model can then determine corrections to account for the deflections that remain. Errors in the modelling will contribute to the uncertainty budget.

3.7.8 *Drift due to environmental effects*

The comparison of NPL, NML and CEM results point to medium- to long-term changes in artefact dimension. While much of this can be related to stabilisation after construction and machining, there is evidence that environmental effects, including humidity, might have an effect on dimension. Although no short term effects have been observed, artefact drift will need to be taken into account if the period between calibration and use is more than a few weeks.

Some of the factors listed above are independent of laboratory, others depend on the calibrating CMM and operating conditions. Based on the results and measurements at NPL, the uncertainty budget presented in Table 6 has been estimated for the full 5 m artefact, that is, an assembly using the two 0.5 m modules and all four 1.0 m modules. The best capability is an estimate of the uncertainty that would pertain if (a) the comparator technique is used to reduce the CMM measurement uncertainty, (b) further tests were performed to confirm or improve the finite element model for deflection during calibration and © any drift is checked before and after the artefact's use. The uncertainty budgets for the other laboratories are similar.

Contributing factor	u (μm)
CMM measurement	5.5
Temperature calibration	0.2
Coefficient of thermal expansion	0.1
Deflection during calibration	0.3
Repeatability of the joins	0.8
Temperature during use	1.3
Deflection during use	1.9
Drift due to environmental effects	3.5
Total uncertainty	7.0
Expanded uncertainty (k = 2)	13.9
Best capability (k = 2)	7.5

Table 6. Uncertainty budget for 5 m assembly based on NPL measurements and operating conditions.

4 VERIFICATION OF CMMS

In order to demonstrate the suitability of the artefact and its associated positioning equipment in the verification of large CMMs, a 3 m assembly (consisting of modules 2, 3 and 4) calibrated by CEM was measured using two large CMMs according to the methods of ISO 10360.

4.1 EXPERIMENT 1

The first experiment was carried out at Renault, Valladolid, a car manufacturer. The CMM had two cantilever arms and a working volume of 6000 mm x 1400 mm x 1700 mm. The 3 m artefact was positioned at an angle of 30° along the four space diagonals of the working volume. The artefact was measured three times in each orientation, with six spheres on each module measured, using five probing points. A Renishaw probe with a 3 mm diameter ball-ended stylus was employed. A summary of the results is given in Figure 5. The graphs show the differences between the distances from the first sphere to the other spheres determined from each of the measurements along four diagonals and the distances determined by the CEM calibration. (These latter distances are not corrected for artefact deflection; the correct distances would differ up to 5 micrometres.) The results show that there are differences of the order of 100 micrometres between the diagonal measurements (independently of any calibration).

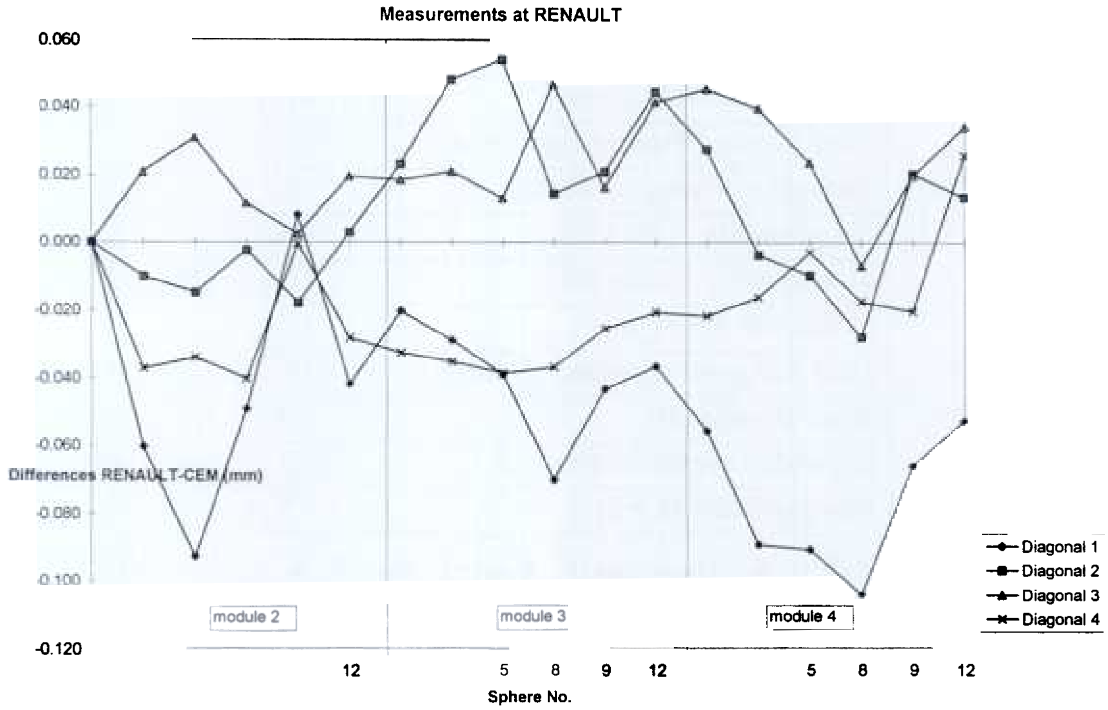


Figure 5. Summary results for Experiment 1. Differences in the estimates of the distances between sphere 1 and other spheres determined by the measurements along 4 diagonals compared with the (uncorrected) distances determined by the CEM calibration.

4.2 EXPERIMENT 2

The second set of measurements were performed at CAF (Construcciones y Auxiliar de Ferrocarriles S.A), Beasain, a railway vehicle manufacturer. The CMM was of gantry type with a measuring volume of 5200 mm x 3000 mm x 1300 mm. The same measurement strategy as in Experiment 1 was employed except that the artefact was positioned at an angle of 25° and only two sets of measurements were taken. Summary results are presented in Figure 6 and are qualitatively similar to those of Experiment 1, only with a larger spread of results (200 micrometres or more).

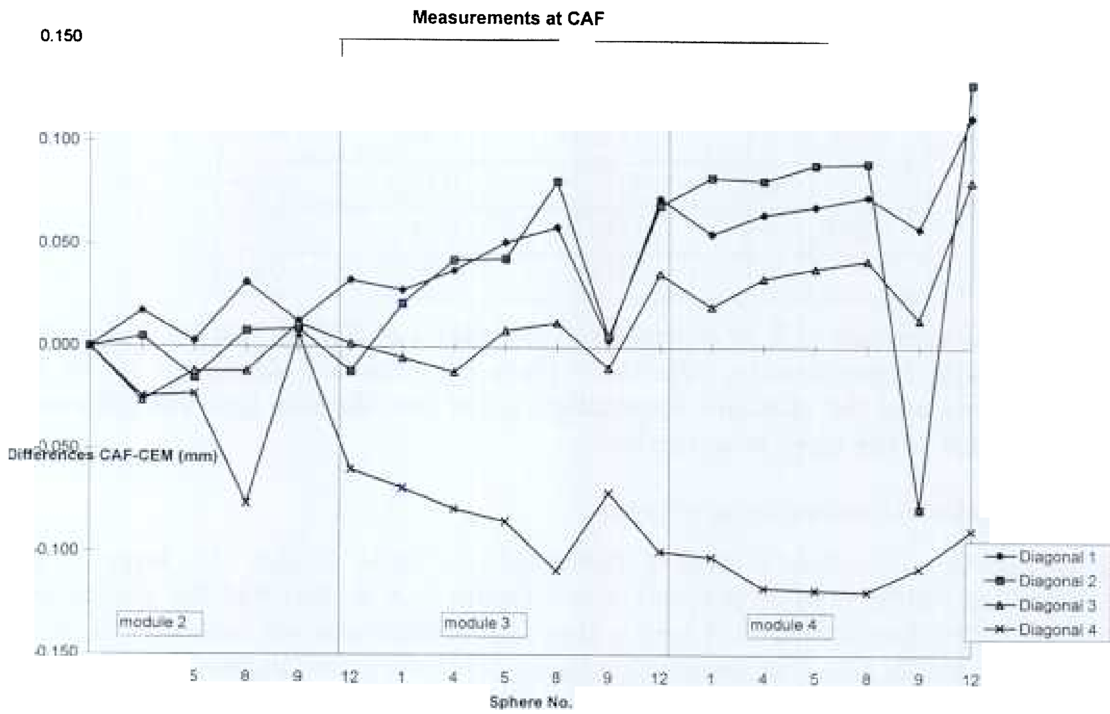


Figure 6. Summary results for Experiment 2. Differences in the estimates of the distances between sphere 1 and other spheres determined by the measurements along 4 diagonals compared with the (uncorrected) distances determined by the CEM calibration.

5 ASSESSMENT OF ARTEFACT DESIGN AND PERFORMANCE

5.1 ANALYSIS OF THE RESULTS

5.1.1 Accuracy of repositioning results

The use of repositioning methods to measure the artefact in itself has introduced no significant uncertainties. An important feature of the analysis of the data is that the mathematical model accurately fits the data with the standard deviation of the residual errors consistently of the order of 0.1 to 0.4 micrometres, comparable with the repeatability of the CMMs being used and well within their statements of length measurement capability.

For example, the standard deviations of the residual errors for all three laboratories associated with the measurement of the three 2 m sections of the artefacts (that is, modules 1 & 2, 2 & 3 and 3 & 4) using one measurement of the join is given in Table 7.

		M12 (μm)	M23 (μm)	M34 (μm)
NPL	*s*	0.41	0.35	0.29
	u	0.48	0.40	0.33
NML	*s*	0.42	0.27	0.26
	u	0.19	0.13	0.16
CEM	*s*	0.15	0.16	0.23
	u	0.15	0.16	0.26

Table 7. Measurement of 2 m assemblies (modules 1 & 2, 2 & 3 and 3 & 4 joined together) using repositioning. The table gives the standard deviation (*s*) of the residual errors and the standard uncertainty (*u*) of the distance between spheres 1 and 20 for each of the three laboratories.

5.1.2 *Accuracy of the calibration information*

Comparing the uncertainty budget presented in Table 1 and the large CMM measurement results presented Figure 5 and Figure 6, it is clear that the uncertainty associated with the calibration is well within the requirements set out in ISO 10360-2 for a length artefact. The only feature that requires further experimentation is the effect of humidity and other environmental factors on long term stability. If humidity is seen to have a significant effect, then the simple process of coating the artefact with a suitable sealant should minimise this effect.

5.1.3 *Position calibration software.*

Another satisfactory aspect is the performance of the position calibration software (Butler *et al.*, 1998). For example, the analysis of the complete set of NPL data relating to all 6 modules involved the solution of 3690 nonlinear equations in 564 unknowns. Because the software exploits the sparsity structure in the resulting matrix equations, the computation time for a complete analysis (consistency checking, initial estimation of the parameters, solution of the non-linear optimisation problem and calculation of all relevant statistics) is of the order of a minute on NPL's DEC Alpha[1] (operating in a multi-user environment). The memory requirements are of the order of 10^4 as opposed to 10^6 for a full matrix approach.

5.2 ARTEFACT USABILITY

A major design aim for the project related to the usability of the artefact. The modularity and low weight of the artefact has led to many benefits. The modules are easily assembled and mounted on the positioning equipment. The positioning equipment can also be used as a purpose-built workbench for assembly. For example, the 3 m artefact used in the large CMM verification was assembled and mounted in 30 minutes by two operators. The complete assembly can be easily manipulated by one person, although it is recommended that two people operate the assembly to guard

[1] The DEC Alpha is comparable in performance to a top-of-the-range Pentium processor.

against accidental damage. The modules are provided with purpose built containers and transported without special instructions to shippers.

5.3 VARIATIONS IN DESIGN

The artefact constructed is only one instance of a generic modular design whose parameters are (a) length, diameter, and thickness of the modules, (b) diameter and thickness of the flanges, and (c) number and locations of the reference surfaces.

The particular choice of parameters was influenced by the requirements of all three laboratories, in particular, the requirement that it would be calibrated on a CMM with longest axis 550 mm. Other requirements would lead to another choice of parameters. Throughout the course of the project other design options have been considered:

- Replacing the titanium mounts with smaller carbon fibre mounts.
- Replacing the steel bolts that hold the flanges together by those made from titanium or aluminium alloy (possibly with their centres bored out).
- Increasing the diameter to increase stiffness.
- Reducing the number of references surfaces. The spacing of the spheres is related to the requirement for repositioning. If CMMs with a working length of 1 m or more are available then only 6 spheres are required on each module (instead of 12).

These changes would significantly reduce the weight and/or increase the stiffness so that the deflections due to gravitational loading would be much reduced.

A simpler artefact can be made from only two modules. The repeatability of the joins is required only if the join has to be reassembled. If only two modules are involved, each module can be calibrated, the modules assembled, the join measured and the two-module assembly used in CMM verification. In this way, a much simpler join can be implemented and the join repeatability errors are eliminated. With this strategy it is possible to calibrate an artefact three or four times the length of the CMM axes. Thus, with a 1 m CMM, it is quite feasible to calibrate accurately a 3 or 4 m artefact.

6 SUMMARY AND CONCLUDING REMARKS

The verification of the length measurement capabilities of large CMMs according to the principles of ISO 10360-2 requires a length artefact that is structurally stable, capable of defining accurately calibrated lengths and easily supported and positioned. This report has described the design, construction and calibration of such an artefact, the main features of which we now summarise.

- The artefact is of modular design, up to five metres long, with each module made from tubular sections of carbon-composite materials. The use of carbon-composite tubular sections has meant that the artefact is light yet rigid. The artefact is easy to transport and store because of its modular construction.

- Each module carries silicon nitride reference spheres spaced uniformly along a measuring line. The calibrated lengths are specified in terms of the distances between these reference spheres.
- The modules have precision ground flanges, also made from carbon fibre, that allow the modules to be joined together semi-kinematically. The operation of these joins is highly repeatable in terms of variations in the distances between pairs of spheres, one on each side of a join.
- An innovative calibration strategy, using repositioning methods and software, has been designed that allows the complete five metre assembly to be calibrated using a CMM with a longest working axis of no more than 500 mm. The modules and the joins of pairs of modules are calibrated in turn. Position calibration software is then used to determine the geometry of the complete assembly.
- The artefact has been calibrated by three metrology laboratories. The expanded uncertainty (k=2) for the length of the five metre assembly for a typical calibration is of the order of 0.010 mm. (The uncertainty depends on the CMM performing the measurements, the operating conditions, etc.) The contribution to this uncertainty budget from the repeatability of the joins is estimated to be no more than 0.002 mm.
- Equipment for supporting and repositioning the artefact during the verification of a CMM has also been designed and constructed.
- The artefact and positioning equipment have been successfully tested in the verification of large CMMs according to the principles of ISO 10360-2.

The artefact constructed reflected the requirements of the project. However, the design philosophy and calibration methodology is flexible and provides a generic template for reference length artefacts. With this template, the verification of the length measuring capabilities of large CMMs can be implemented in practice.

ACKNOWLEDGEMENTS

The work reported on here was jointly funded by the European Commission Directorate-General XII, Measurement and Testing (contract MAT1-CT94-0012) and the Department of Trade and Industry's National Measurement System Policy Unit Programme for Length. The contribution of the project team members at NPL, NML, CEM and Tekniker is gratefully acknowledged. We are also extremely grateful to Renault, Valladolid and CAF, Beasain, for their valuable co-operation during the CMM verification experiments. We thank B J Chorley, NPL, for careful reviews of drafts of this report.

7 REFERENCES

Butler, B. P., Forbes, A. B. and Kenward, P. K., (1998): Position Calibration Software: Fortran implementation of algorithms to determine frame of reference from measurements of registration points. *NPL Report CISE 14/98,* National Physical Laboratory, Teddington.

Cox, M. G., Cross, N. R., Flack, D. R., Forbes, A. B. and Peggs, G. N. (1997): Measurement of artefacts using repositioning methods. *NPL Report CLM 2,* National Physical Laboratory, Teddington.

Cox, M. G., Forbes, A. B., Harris, P. M., and Peggs, G. N. (1998): Determining CMM behaviour from measurements of standard artefacts. *NPL Report CISE 15/98,* National Physical Laboratory, Teddington.

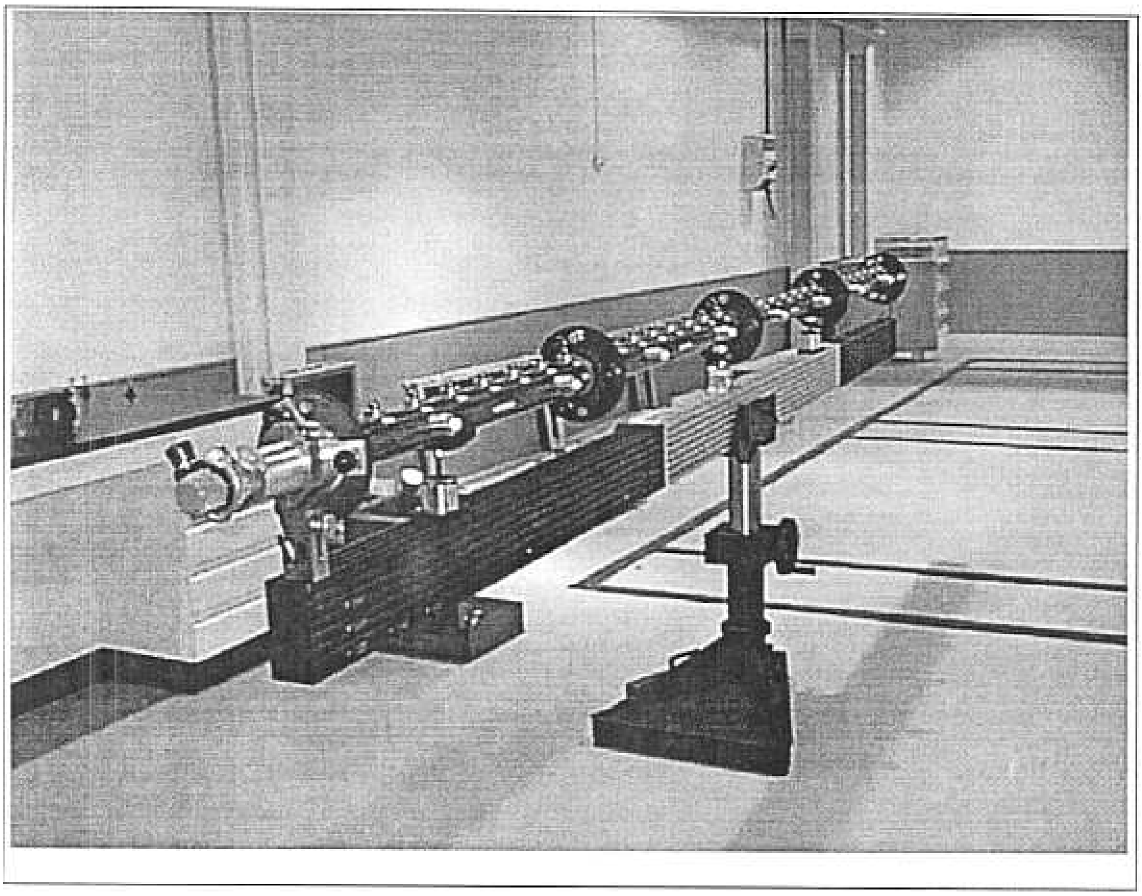
Cox, M. G., Forbes, A. B. and Peggs, G. N. (1997): CMM verification and grading. *NPL Report CLM 1,* National Physical Laboratory, Teddington.

Forbes, A. B., (1989): Least squares best fit geometric elements. *NPL Report DITC 140/89,* National Physical Laboratory, Teddington.

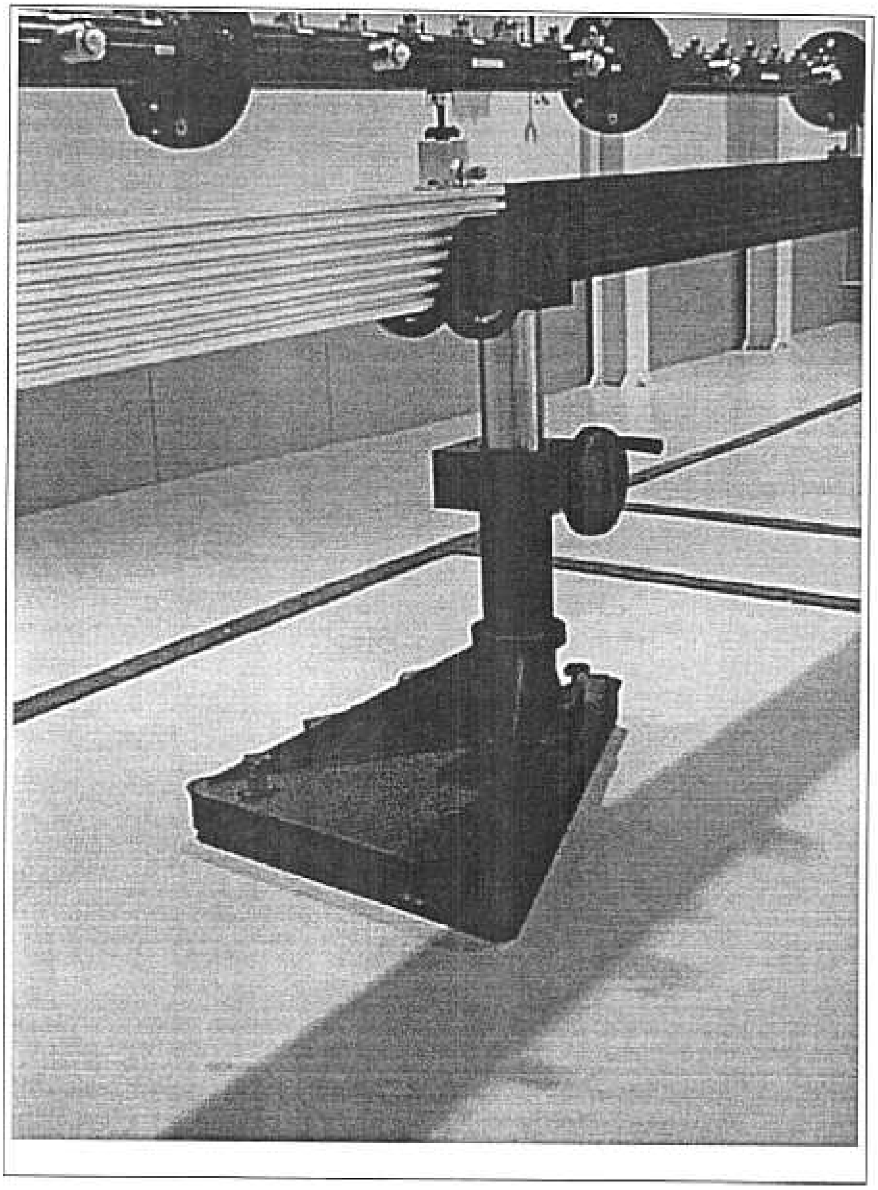
ISO (1995): BS EN ISO 10360-2. *Co-ordinate metrology part 2: performance assessment of co-ordinate measuring machines.*

ANNEXES

1 POSITIONING EQUIPMENT



Four metre artefact assembly mounted on the positioning equipment. The alignment telescope is seen attached to the near end of the artefact assembly.



Central pillar of the positioning equipment. The artefact can be raised or lowered using the telescopic mechanism. The supporting beam pivots to allow the artefact to be set at an angle. Also seen is the micrometer screw jack used to restore the artefact to a known deflection.

2 FINITE ELEMENT CORRECTIONS

2.1 DEFLECTIONS FOR A 1 METRE MODULE

The following table gives, in columns 2 to 4, the deflections in each co-ordinate at each sphere location (indexed by the first column) calculated by the finite element module for a 1 m module in standard position supported under spheres 2 and 7. The units are in micrometres.

1	-0.3700	0	-0.7900
2	-0.2200	0	0
3	-0.0600	0	0.2200
4	-0.0100	0	0.2600
5	0.0100	0	0.2500
6	0.0600	0	0.2100
7	0.2000	0	0
8	0.3300	0	-0.6400
9	0	0	-0.7900
10	0	0	0.2600
11	0	0	0.2100
12	0	0	0

2.2 DEFLECTIONS FOR A 2 METRE ASSEMBLY

The following table gives, in columns 2 to 4, the deflections in each co-ordinate at each sphere location (indexed by the first column) calculated by the finite element module for an assembly of 2 m modules in standard position supported under spheres 4 (sphere 4 of module 1) and 17 (sphere 5 of module 2). The units are in micrometres.

1	-0.9500	0	-3.7600
2	-0.8000	0	-1.8500
3	-0.4400	0	-0.4300
4	0.2000	0	0
5	0.8000	0	-1.2600
6	1.0200	0	-3.3200
7	0.9400	0	-5.5100
8	0.6300	0	-7.2600
9	0	0	-3.8700
10	0	0	-0.3500
11	0	0	-1.3300
12	0	0	-5.4800
13	-0.5900	0	-7.3100
14	-0.9200	0	-5.6300
15	-1.0200	0	-3.4700
16	-0.8300	0	-1.3700
17	-0.2900	0	0
18	0.3100	0	-0.1800
19	0.6400	0	-1.2900
20	0.7700	0	-2.8400
21	0	0	-7.3400
22	0	0	-3.4700
23	0	0	0
24	0	0	-1.3300

2.3 DEFLECTIONS FOR A 3 METRE ASSEMBLY

The following table gives the differences from nominal of selected inter-sphere distances for a 3 m module assembly supported during its use in CMM verification at an angle of 25°. These differences arise due to deflections caused by gravitational loading. The first two columns give the index of the spheres, the third column gives the nominal distance in m and the fourth column gives the correction in micrometres.

1	32	2.8750	3.8000
1	13	1.0000	0.9000
1	25	2.0000	-0.6000
4	16	1.0000	0.8000
4	28	2.0000	2.9000
5	17	1.0000	-0.6000
5	29	2.0000	3.5000
8	20	1.0000	-2.4000
8	32	2.0000	3.7000
13	25	1.0000	-1.5000
16	28	1.0000	2.1000
17	29	1.0000	4.1000
20	32	1.0000	6.1000

3 MEASUREMENT RESULTS FOR 1 METRE MODULES

The following three sets of tables give, in columns 2 to 4, the differences between the estimates of the reference sphere centre co-ordinates (indexed by the first column) and their nominal values. The units are in millimetres.

3.1 NPL RESULTS

3.1.1 Module 1

1	0	0	0
2	-0.0096	0.1083	0.0480
3	0.1283	-0.0120	-0.0548
4	0.0361	0.1058	-0.0382
5	0.1576	-0.1184	0.0880
6	0.1817	0.1244	-0.0489
7	0.0563	0.1831	0.0477
8	0.1492	0.4822	-0.0611
9	0.0183	0.8401	-0.0291
10	0.0373	-0.7158	0.0561
11	-0.1608	1.1265	-0.0079
12	-0.0883	-0.4023	0.1082

3.1.2 Module 2

1	0	0	0
2	0.0777	-0.1001	0.0054
3	-0.0897	0.0778	0.1031
4	-0.0474	0.2322	0.0556
5	-0.0995	0.2875	0.0394
6	-0.0811	0.1111	0.0292
7	0.0671	0.4770	0.0590
8	0.0819	0.4536	-0.1180
9	-0.0657	1.1395	-0.1493
10	0.3103	-0.4612	-0.5556
11	0.3388	1.2431	-0.9405
12	0.0690	-0.4024	-0.1678

3.1.3 Module 3

1	0	0	0
2	0.1782	0.1793	-0.0880
3	0.0830	0.0095	-0.1076
4	0.0508	0.3058	-0.0075
5	0.0094	0.2674	-0.0511
	-0.0154	0.2411	-0.0785
	0.0785	0.3697	0.0300
	-0.0346	0.2346	-0.0198
9	-0.0166	0.7980	-0.2319
10	0.0888	-1.0096	-0.1692
11	-0.0588	1.5736	-0.2585
12	0.0672	-0.6626	-0.3722

3.1.4 Module 4

1	0	0	0
2	-0.0661	0.6762	-0.0754
3	0.0809	0.3765	0.0174
4	0.1549	0.2861	-0.1138
5	-0.1274	0.5107	-0.1556
6	-0.0469	0.6520	-0.0889
7	-0.1138	1.1086	-0.2229
8	0.0268	0.5757	-0.2485
9	-0.0364	1.6281	-0.6324
10	0.0624	-0.7374	-0.3941
11	0.0694	2.0255	-0.2815
12	0.2373	-0.5195	-0.3713

3.2 NML RESULTS

3.2.1 Module 1

1	0	0	0
2	-0.0083	0.1083	0.0497
3	0.1306	-0.0123	-0.0527
4	0.0396	0.1056	-0.0361
5	0.1624	-0.1190	0.0903
6	0.1881	0.1242	-0.0470
7	0.0642	0.1821	0.0485
8	0.1590	0.4822	-0.0619
9	0.0176	0.8409	-0.0311
10	0.0407	-0.7187	0.0572
11	-0.1578	1.1281	-0.0052
12	-0.0803	-0.4041	0.1072

3.2.2 Module 2

1	0	0	0
2	0.0790	-0.1036	0.0086
3	-0.0870	0.0768	0.1043
4	-0.0437	0.2317	0.0575
5	-0.0952	0.2867	0.0415
6	-0.0748	0.1108	0.0314
7	0.0748	0.4762	0.0606
8	0.0914	0.4533	-0.1186
9	-0.0645	1.1412	-0.1496
10	0.3142	-0.4626	-0.5546
11	0.3442	1.2447	-0.9398
12	0.0772	-0.4047	-0.1668

3.2.3 *Module 3*

1	0	0	0
2	0.1797	0.1782	-0.0881
3	0.0853	0.0080	-0.1069
4	0.0542	0.3040	-0.0068
5	0.0138	0.2653	-0.0503
6	-0.0090	0.2400	-0.0776
7	0.0866	0.3679	0.0298
8	-0.0249	0.2343	-0.0204
9	-0.0151	0.7984	-0.2351
10	0.0933	-1.0139	-0.1692
11	-0.0541	1.5738	-0.2585
12	0.0765	-0.6647	-0.3759

3.2.4 *Module 4*

	0	0	0
	-0.0653	0.6760	-0.0761
3	0.0815	0.3760	0.0162
4	0.1561	0.2860	-0.1153
	-0.1266	0.5101	-0.1567
	-0.0450	0.6518	-0.0893
	-0.1112	1.1080	-0.2238
8	0.0306	0.5762	-0.2488
9	-0.0362	1.6290	-0.6336
10	0.0640	-0.7392	-0.3963
11	0.0706	2.0261	-0.2839
12	0.2411	-0.5207	-0.3730

3.3 CEM RESULTS

3.3.1 *Module 1*

	0	0	0
2	-0.0079	0.1074	0.0497
3	0.1311	-0.0135	-0.0536
4	0.0407	0.1046	-0.0373
5	0.1636	-0.1194	0.0889
6	0.1891	0.1238	-0.0485
7	0.0649	0.1821	0.0482
8	0.1593	0.4828	-0.0615
9	0.0184	0.8428	-0.0329
10	0.0408	-0.7209	0.0559
11	-0.1553	1.1304	-0.0081
12	-0.0792	-0.4048	0.1054

3.3.2 Module 2

1	0	0	0
2	0.0790	-0.1038	0.0075
3	0.0867	0.0763	0.1022
4	0.0433	0.2315	0.0551
5	-0.0943	0.2874	0.0388
6	-0.0743	0.1112	0.0294
7	0.0753	0.4766	0.0596
8	0.0917	0.4537	-0.1185
9	-0.0658	1.1434	-0.1523
10	0.3137	-0.4654	-0.5580
11	0.3450	1.2480	-0.9442
12	0.0779	-0.4061	-0.1702

3.3.3 Module 3

1	0	0	0
2	0.1797	0.1777	-0.0889
3	0.0858	0.0072	-0.1083
4	0.0549	0.3033	-0.0080
5	0.0151	0.2652	-0.0517
6	-0.0078	0.2395	-0.0793
7	0.0878	0.3676	0.0288
8	-0.0240	0.2347	-0.0200
9	-0.0165	0.7999	-0.2379
10	0.0931	-1.0168	-0.1710
11	-0.0533	1.5768	-0.2613
12	0.0780	-0.6662	-0.3789

3.3.4 Module 4

1	0	0	0
2	-0.0654	0.6759	-0.0774
3	0.0813	0.3759	0.0136
4	0.1554	0.2863	-0.1180
5	-0.1271	0.5111	-0.1593
6	-0.0461	0.6528	-0.0917
7	-0.1130	1.1095	-0.2253
8	0.0282	0.5780	-0.2494
9	-0.0376	1.6325	-0.6354
10	0.0618	-0.7404	-0.4012
11	0.0699	2.0295	-0.2891
12	0.2392	-0.5223	-0.3743

3.4 INTERCOMPARISON OF RESULTS IN TERMS OF DISTANCES

The following tables give the difference between the distances between the spheres indexed by columns 1 and 2 as measured by NPL, NML and CEM. Columns 3, 4 and 5 give (NPL-NML), (NML-CEM) and (CEM-NPL) respectively. The units are millimetres.

3.4.1 *Module 1*

1	2	-0.0012	-0.0004	0.0016
1	3	-0.0023	-0.0005	0.0028
1	4	-0.0034	-0.0011	0.0045
1	5	-0.0048	-0.0012	0.0060
1	6	-0.0064	-0.0009	0.0073
1	7	-0.0078	-0.0007	0.0086
1	8	-0.0099	-0.0003	0.0102
2	8	-0.0086	0.0001	0.0086
3	8	-0.0076	0.0002	0.0074
4	8	-0.0064	0.0008	0.0056
5	8	-0.0051	0.0009	0.0041
6	8	-0.0035	0.0006	0.0029
7	8	-0.0020	0.0004	0.0016

3.4.2 *Module 2*

1	2	-0.0013	0.0000	0.0012
1	3	-0.0027	-0.0003	0.0030
1	4	-0.0037	-0.0004	0.0041
1	5	-0.0044	-0.0009	0.0053
1	6	-0.0062	-0.0005	0.0068
1	7	-0.0076	-0.0005	0.0081
1	8	-0.0095	-0.0002	0.0098
2	8	-0.0083	-0.0003	0.0086
3	8	-0.0068	0.0000	0.0068
4	8	-0.0058	0.0002	0.0056
5	8	-0.0052	0.0007	0.0045
6	8	-0.0033	0.0003	0.0030
7	8	-0.0019	0.0003	0.0016

3.4.3 Module 3

1	2	-0.0015	0.0000	0.0015
1	3	-0.0023	-0.0004	0.0028
1	4	-0.0035	-0.0007	0.0042
1	5	-0.0045	-0.0012	0.0057
1	6	-0.0065	-0.0012	0.0076
1	7	-0.0081	-0.0012	0.0092
1	8	-0.0098	-0.0009	0.0107
2	8	-0.0082	-0.0009	0.0091
3	8	-0.0074	-0.0005	0.0079
4	8	-0.0063	-0.0002	0.0065
5	8	-0.0053	0.0003	0.0050
6	8	-0.0033	0.0003	0.0030
7	8	-0.0017	0.0003	0.0014

3.4.4 Module 4

1	2	-0.0008	0.0001	0.0007
1	3	-0.0006	0.0002	0.0004
1	4	-0.0012	0.0007	0.0005
1	5	-0.0008	0.0005	0.0004
1	6	-0.0019	0.0012	0.0008
1	7	-0.0026	0.0018	0.0008
1	8	-0.0037	0.0024	0.0013
2	8	-0.0029	0.0023	0.0007
3	8	-0.0032	0.0022	0.0010
4	8	-0.0026	0.0017	0.0009
5	8	-0.0029	0.0019	0.0010
6	8	-0.0018	0.0012	0.0006
7	8	-0.0011	0.0006	0.0006

3.4.5 Module 1 re-measured by NPL

1	2	-0.0002	-0.0004	0.0006
1	3	-0.0004	-0.0005	0.0009
1	4	-0.0004	-0.0011	0.0015
1	5	-0.0003	-0.0012	0.0016
1	6	-0.0012	-0.0009	0.0021
1	7	-0.0016	-0.0007	0.0023
1	8	-0.0023	-0.0003	0.0026
2	8	-0.0021	0.0001	0.0020
3	4	0.0000	-0.0006	0.0006
3	8	-0.0019	0.0002	0.0017
4	5	0.0001	-0.0001	0.0001
4	8	-0.0019	0.0008	0.0011
5	8	-0.0019	0.0009	0.0010
6	8	-0.0011	0.0006	0.0005
7	8	-0.0007	0.0004	0.0003

4 MEASUREMENTS OF JOINS

The following three sets of tables give, in columns 2 to 4, the differences between the estimates of the reference sphere centres co-ordinates (indexed by the first column) and their nominal values. The units are in millimetres.

4.1 NPL RESULTS

4.1.1 Join 1 and 2

7	0	0	0
8	0.0934	0.0938	-0.1613
13	0.0160	-0.2629	0.1416
14	0.0935	-0.5571	0.1153
12	-0.2919	-0.6180	0.1055
21	0.0503	0.7746	-0.1486

4.1.2 Join 2 and 3

7	0	0	0
8	0.0151	-0.2247	-0.1472
13	-0.1479	-0.5846	0.0075
14	0.0305	-0.6491	-0.0615
12	-0.1068	-0.9556	-0.1234
21	-0.0096	0.2715	-0.1455

4.1.3 Join 3 and 4

7	0	0	0
8	-0.1143	-0.5641	-0.0677
13	0.0482	-0.9412	0.0589
14	-0.0164	-0.7070	-0.0245
12	-0.2800	-1.1274	-0.2705
21	0.2788	0.6748	-0.5900

4.2 NML RESULTS

4.2.1 Join 1 and 2

5	0	0	0
6	0.0252	0.2204	-0.1896
7	-0.0994	0.2559	-0.1453
8	-0.0053	0.5328	-0.3068
13	-0.0810	0.3643	0.0095
14	-0.0018	0.2536	-0.0167
15	-0.1672	0.4266	0.0438
16	-0.1236	0.5745	-0.0376
11	-0.3289	1.1544	-0.2235
12	-0.2809	-0.4228	0.0416
21	-0.1601	1.3427	-0.3633
22	0.2122	-0.2773	-0.3923

4.2.2 Join 2 and 3

5	0	0	0
6	0.0196	0.1210	-0.0313
7	0.1685	0.2996	-0.0222
8	0.1843	0.3314	-0.2208
13	0.0242	0.2288	-0.1148
14	0.2040	0.4210	-0.2348
15	0.1105	0.2640	-0.2868
16	0.0799	0.5735	-0.2185
11	0.3968	1.0138	-0.9059
12	0.1954	0.5268	-0.3247
21	-0.0153	1.2150	-0.0912
22	0.1111	0.5705	-0.6074

4.2.3 Join 3 and 4

5	0	0	0
6	-0.0236	0.0110	-0.0608
7	0.0715	0.1314	0.0136
8	-0.0407	0.0116	-0.0681
13	0.1243	0.0308	0.0803
14	0.0592	0.7086	-0.0176
15	0.2066	0.4095	0.0532
16	0.2813	0.3213	-0.0987
11	-0.0908	1.2211	-0.3291
12	0.0560	0.9891	-0.2707
21	0.0759	1.6775	-0.5297
22	0.1799	0.6884	-0.3827

4.3 CEM RESULTS

4.3.1 Join 1 and 2

5	0	0	0
6	0.0254	0.2211	-0.1897
	-0.0989	0.2580	-0.1452
8	-0.0043	0.5363	-0.3058
13	-0.0768	0.3625	0.0090
14	0.0024	0.2528	-0.0171
15	-0.1634	0.4265	0.0433
16	-0.1198	0.5753	-0.0381
11	-0.3281	1.1548	-0.2281
12	-0.2782	0.4232	0.0421
21	-0.1545	1.3452	-0.3633
22	0.2180	0.2762	-0.3967

4.3.2 *Join 2 and 3*

5	0	0	0
6	0.0198	-0.1222	-0.0323
7	0.1691	0.2979	-0.0242
8	0.1854	0.3294	-0.2238
13	0.0278	0.2286	-0.1154
14	0.2074	0.4206	-0.2362
15	0.1135	0.2636	-0.2882
16	0.0827	0.5729	-0.2198
11	0.3966	1.0150	-0.9093
12	0.1953	-0.5306	-0.3275
21	-0.0121	1.2168	-0.0940
22	0.1144	-0.5720	-0.6103

4.3.3 *Join 3 and 4*

5	0	0	0
6	-0.0230	-0.0172	-0.0587
7	0.0725	0.1195	0.0183
8	-0.0394	-0.0057	-0.0603
13	0.1270	0.0436	0.0743
14	0.0620	0.7156	-0.0198
15	0.2092	0.4107	0.0555
16	0.2836	0.3164	-0.0930
11	-0.0871	1.2309	-0.3210
12	0.0547	-0.9946	-0.2778
21	0.0838	1.6871	-0.5456
22	0.1792	-0.6938	-0.3746

4.4 COMPARISON OF JOIN RESULTS

The following tables give the differences in the distances between pairs of spheres centres. Each row gives (j, l, $d_{NPL}(j,l) - d_{NML}(j,l)$, $d_{NML}(j,l) - d_{CEM}(j,l)$, $d_{CEM}(j,l) - d_{NPL}(j,l)$).

4.4.1 *Join 1 and 2*

7	8	-0.0010	-0.0005	0.0016
7	13	-0.0024	-0.0037	0.0061
7	14	-0.0037	-0.0037	0.0074
8	13	-0.0014	-0.0032	0.0045
8	14	-0.0027	-0.0032	0.0059
13	14	-0.0013	0.0000	0.0013

4.4.2 *Join 2 and 3*

7	8	-0.0006	-0.0004	0.0011
7	13	-0.0030	-0.0029	0.0059
	14	-0.0045	-0.0028	0.0073
8	13	-0.0023	-0.0025	0.0048
8	14	-0.0039	-0.0024	0.0062
13	14	-0.0016	0.0001	0.0014

4.4.3 Join 3 and 4

7	8	-0.0009	-0.0003	0.0012
7	13	-0.0028	-0.0017	0.0045
7	14	-0.0039	-0.0018	0.0058
8	13	-0.0019	-0.0014	0.0033
8	14	-0.0030	-0.0016	0.0045
13	14	-0.0012	-0.0001	0.0013

4.5 REPEATABILITY OF JOIN 2 AND 3 FROM CEM RESULTS

Join 2 & 3 was measured six times by CEM. For each set of measurements, we used the position calibration software to combine the measurements of the join with the measurements of the individual modules to determine six estimates of the reference spheres along the 2 m assembly. From these estimates we have calculated six estimates of the distance between pairs of points and the standard deviations of these estimates of the distances. Below are these standard deviations, firstly for all six sets, secondly omitting the third set. The units are in micrometres.

4.5.1 Standard deviations of inter-sphere distances - all six measurements

1	5	0.0516
1	6	0.0275
1	7	0.0080
1	8	0.0105
1	13	0.7859
1	14	0.7886
1	15	0.7990
1	16	0.7993
1	17	0.7906
1	18	0.7907
1	19	0.7907
1	20	0.7910
2	20	0.7913
3	20	0.7912
4	20	0.7908
5	20	0.8416
6	20	0.8178
7	20	0.7886
8	20	0.7816
13	20	0.0127
14	20	0.0063
15	20	0.0130
16	20	0.0111

4.5.2 *Standard deviations of inter-sphere distances - omitting the 3rd set of measurements*

1	5	0.0223
1	6	0.0183
1	7	0.0090
1	8	0.0103
1	13	0.4128
1	14	0.4082
1	15	0.4056
1	16	0.4051
1	17	0.4065
1	18	0.4069
1	19	0.4066
1	20	0.4073
2	20	0.4073
3	20	0.4076
4	20	0.4076
5	20	0.4287
6	20	0.4249
7	20	0.4050
8	20	0.3985
13	20	0.0107
14	20	0.0062
15	20	0.0096
16	20	0.0042

5 MEASUREMENT OF 4 METRE ASSEMBLY

The following tables give, in column 3, the estimates of the inter-sphere distances (indexed by columns 1 and 2) along with estimates of their standard deviation in column 4. The units are in millimetres.

5.1 NPL RESULTS

1	44	3875.14545	0.00147
1	13	1000.07223	0.00079
1	25	1999.99118	0.00105
1	37	3000.11929	0.00127
4	16	999.98807	0.00078
4	28	2000.00571	0.00104
4	40	3000.23763	0.00126
5	17	999.81466	0.00078
5	29	1999.84290	0.00104
5	41	2999.83402	0.00126
8	20	1000.00438	0.00072
8	32	1999.80681	0.00101
8	44	2999.99605	0.00127
13	25	999.91895	0.00072
13	27	1250.00168	0.00085
16	28	1000.01768	0.00078
16	40	2000.24964	0.00105
17	29	1000.02835	0.00078
17	41	2000.01948	0.00105
20	32	999.80251	0.00072
20	44	1999.99173	0.00105
25	37	1000.12814	0.00073
28	40	1000.23199	0.00078
29	41	999.99113	0.00078
32	44	1000.18927	0.00079

5.2 NML RESULTS

1	44	3875.18178	0.00081
1	13	1000.08208	0.00044
1	25	2000.01200	0.00051
1	37	3000.15101	0.00062
4	16	999.99931	0.00044
4	28	2000.02757	0.00051
4	40	3000.26821	0.00062
5	17	999.82471	0.00042
5	29	1999.86369	0.00050
5	41	2999.86200	0.00078
8	20	1000.01401	0.00042
8	32	1999.82797	0.00050
8	44	3000.02301	0.00078
13	25	999.92993	0.00041
13	27	1250.01574	0.00041
16	28	1000.02830	0.00041
16	40	2000.26898	0.00054
17	29	1000.03909	0.00041
17	41	2000.03742	0.00073
20	32	999.81403	0.00041
20	44	2000.00906	0.00073
25	37	1000.13905	0.00047
28	40	1000.24071	0.00047
29	41	999.99833	0.00068
32	44	1000.19507	0.00068

5.3 CEM RESULTS

1	44	3875.19108	0.00056
1	13	1000.08675	0.00034
1	25	2000.02034	0.00038
1	37	3000.16254	0.00047
4	16	1000.00302	0.00034
4	28	2000.03476	0.00038
4	40	3000.27809	0.00047
5	17	999.82914	0.00029
5	29	1999.87191	0.00037
5	41	2999.87234	0.00052
8	20	1000.01891	0.00029
8	32	1999.83677	0.00037
8	44	3000.03171	0.00052
13	25	999.93360	0.00028
13	27	1250.01935	0.00028
16	28	1000.03179	0.00028
16	40	2000.27514	0.00039
17	29	1000.04287	0.00031
17	41	2000.04332	0.00048
20	32	999.81793	0.00031
20	44	2000.01284	0.00048
25	37	1000.14223	0.00034
28	40	1000.24339	0.00034
29	41	1000.00045	0.00045
32	44	1000.19496	0.00045