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Measurement of artefacts using repositioning methods

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ABSTRACT
This report describes a methodology for the measurement of large or complex artefacts by co-ordinate measuring machines (CMMs) using repositioning. The approach is applicable to measurement tasks where it is not possible or practical to measure all relevant parts of the workpiece surfaces without repositioning the artefact. By tracking the position of three or more registration points defined by reference features (sphere centres, for example) that move rigidly with the workpiece it is possible to relate measurements taken in different positions to one frame of reference. Practical aspects of implementing the methodology are discussed and the results of measurement experiments presented. The method provides an effective way of extending significantly the measuring capability of a CMM.

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1 INTRODUCTION

A problem that can arise in the calibration of large or complex artefacts by a co-ordinate measuring machine (CMM) is that not all of the relevant surfaces of the artefact can be probed without moving the artefact (or employing special probe configurations that introduce significant probing errors). For example in the measurement of shell-like shapes it is usually easy to probe each of the inner and outer surfaces individually, but much more difficult to position the shell in order to probe both the inner and outer surfaces in an unbroken sequence without recourse to changing the probe or using a complex probing arrangement. A second example is the calibration of a step gauge or similar length artefact which is longer than the longest diagonal of the CMM. It may be possible to measure accurately all the step faces on the gauge by repositioning the artefact so that they lie within the working volume of the CMM. In both these examples, measurements taken in different positions lie in different frames of reference and in order to combine these measurements it is necessary to establish an accurate link between the frames of reference. This report describes a methodology to provide such links.

The remainder of this report is organised as follows. In section 2, we discuss the general principles of the repositioning methodology, while section 3 considers its practical implementation. Section 4 reports on three measurement problems which were solved using repositioning methods. Our concluding remarks are given in section 5.

2 GENERAL PRINCIPLES OF REPOSITIONING METHODS

2 MATHEMATICAL MODEL FOR REPOSITIONING

The basic fact that is required for repositioning is that the location of three (non-collinear) points are necessary and sufficient to determine a frame of reference. Thus, if we move an artefact from position to position but keep track of the positions of three or more points that move rigidly with the artefact (*registration points*) it will be possible to relate the different positions of the artefact to a common frame of reference. (Section 3.1 considers how registration points can be determined in practice.) The mathematical model associated with repositioning is considered in detail in Butler, Forbes and Kenward (1997). Here we give an overview. We consider first the problem of relating two frames of reference to each other.

Let $\{y_j, j \in S\}$ represent registration points in some frame of reference. We suppose that these registration points move rigidly with the artefact to be measured. Let $\{\hat{y}_j, j \in S\}$ denote their location in a subsequent position. Suppose $\{x_j, j \in S\}$ and $\{\hat{x}_j, j \in S\}$ denote a set of measurements of the registration points in the two positions with associated measurement errors $\{\epsilon_j, j \in S\}$ and $\{\hat{\epsilon}_j, j \in S\}$, respectively. Similarly, let $\{w_i, i \in I\}$ and $\{\hat{w}_i, i \in I\}$ represent measurements of the artefact's surface in the two positions.

The basic model equations arise from the fact that the positions of the registration points in the subsequent frame of reference are given by

$$\hat{y}_j = T(y_j, t), \quad j \in S,$$

where T is a *rototranslation*, depending on parameters $\boldsymbol{t} = (\boldsymbol{a}^{\mathrm{T}}, \theta^{\mathrm{T}})^{\mathrm{T}}$, of the form

$$T(\boldsymbol{x}, \boldsymbol{t}) = R(\theta) R_0(\boldsymbol{x} - \boldsymbol{a}),$$

i.e., a transformation composed of a translation defined by parameters $\boldsymbol{a}$, a rotation matrix defined by three rotation angles θ, and a fixed rotation matrix R_0.

Adopting the notation set out above, the model equations for the two-position case take the form

$$\begin{aligned} \boldsymbol{x}_i &= \boldsymbol{y}_j + \epsilon_j, \\ \hat{\boldsymbol{x}}_j &= \hat{\boldsymbol{y}}_j + \hat{\epsilon}_j, \quad j \in S. \end{aligned}$$

If we assume that the components of the errors ϵ_i are uncorrelated and normally distributed then an efficient, unbiased estimate of the parameters can be determined by minimising

$$\sum\nolimits_{j \in S} \left(\epsilon_j^{\mathrm{T}} \epsilon_j + \hat{\epsilon}_j^{\mathrm{T}} \hat{\epsilon}_j \right)$$

with respect to the parameters $\boldsymbol{y}_j$ and $\boldsymbol{t}$. Once the optimal transformation parameters have been determined, the measurements taken in the subsequent frame of reference can be transformed into the initial frame by the inverse transformation:

$$\check{\boldsymbol{w}}_i = T^{-1}(\hat{\boldsymbol{w}}_i, \boldsymbol{t}).$$

This model can be generalised to cater for an arbitrary number of subsequent frames of reference, within which different subsets of the registration points are measured. (From the modelling point of view, there is some arbitrariness in specifying which frame of reference is the initial one; in actual measurement experiments it is usual to index the frames of reference chronologically; from a computational point of view there are advantages to assign low indices to positions in which a large number of registration points are measured.)

2.2 EXPERIMENTAL DESIGN FOR REPOSITIONING

2.2.1 Identifiable experimental designs

Given three or more (non-collinear) points measured in two frames of reference it is possible to determine the transformation relating one frame to the other using the singular value decomposition (SVD); see, for example, Golub (1996), Golub and Van Loan (1996, Chapter 12), Forbes (1992), Butler, Forbes and Kenward (1997)). Using this approach it is possible to determine estimates of the transformation parameters linking multiple frames of reference. Suppose there are n registration points $\boldsymbol{y}_j$ and let I_k be the index set of all the registration points measured in the kth position. To determine all the transformation parameters, it is sufficient to find an ordering i_k of the index sets I_k such that I_{i_k} has at least three points in general position in common with the union of I_{j_k}, $j_k < i_k$. We will call an experimental design for which this property holds *identifiable*.

A sufficient condition for a measurement design to be identifiable is that at least three registration points measured in a frame of reference are measured in the next frame of reference. We term such designs *sequentially identifiable*. This condition is not a necessary condition and there are many identifiable designs for which this condition does not hold.

Example. Suppose the index sets are: $I_0 = \{1,2,3,4\}$, $I_1 = \{2,3,4,5\}$ and $I_2 = \{1,4,5,6\}$. Since I_0 and I_1 have three points in common it should be possible to relate the two frames of reference and give an estimate of the position of y_5 in the first frame of reference. Since I_2 and $I_0 \cup I_1$ have three points in common it is possible to relate the third frame of reference to the first and determine an estimate of y_6.

On the other hand, if the index sets are: $I_0 = \{1,2,3,4\}$, $I_1 = \{3,4,5,6\}$ and $I_2 = \{1,2,5,6\}$, there is no pair of sets with three points in common and the method described above cannot be implemented. This does not mean that the transformation parameters are not well-determined by the data, only that the initial estimation procedure based on matching sets of points using the SVD cannot be used.

The problem of determining whether or not a measurement strategy leads to an identifiable system is purely a combinatorial problem and a solution algorithm is described in Butler, Forbes and Kenward (1997). Below, we consider other aspects of experimental design.

2.2.2 *Placement of the registration points*

While determining if a measurement strategy is identifiable is a combinatorial problem depending only on the index sets associated with the strategy, the accuracy of the method depends directly on the geometry of the registration points. We will assume that a sequentially identifiable strategy has been adopted. What follows, however, applies equally well with appropriate modifications to other strategies. Two situations need to be avoided. The first is where the registration points linking one frame of reference to the subsequent frame lie close to a straight line. In this case the rotation about this axis will be poorly determined. The second is where the registration points linking one frame of reference to the subsequent frame occupy a small volume compared to the artefact surfaces being measured. In this case small measurement errors in determining the registration points will be magnified when applying the transformations to map the surface measurements to a single frame of reference.

2.2.3 *Example: translation along an axis*

Consider ten registration points lying approximately on a 5×2 grid as shown below:

1	3	5	7	9
2	4	6	8	10

Then measuring registration points 1 to 6 in the initial frame, 3 to 8 in the first frame and 5 to 10 in the second frame yields a sequentially identifiable strategy associated with translating the artefact along an axis. An advantage of using symmetrical patterns is that software to control the

CMM to perform the measurements can be written more efficiently. The size of the grid is determined by what the CMM can measure in one position.

2.2.4 Example: rotation along an axis

Suppose six registration points are placed at the corners of a hexagon and numbered sequentially. Measuring points 1, 2, 3 and 4, then 2, 3, 4 and 5, and so on until 6, 1, 2 and 3 are measured provides a sequentially identifiable strategy which can be used in rotating an artefact about an axis approximately orthogonal to the plane of the hexagon.

Example: measuring a cylindrical artefact

The translation and rotation approach can be combined for measuring a cylindrical artefact. Consider 15 registration points at the corners of five equilateral triangles positioned approximately orthogonally to a cylinder axis separated by roughly equal distances so that the registration points lie approximately on three generators of the cylinder. The numbering of the registration points can be inferred from the diagram below.

1	4	7	10	13
2	5	8	11	14
3	6	9	12	15

Table 1 gives a measurement strategy which allows for translations and rotations of the cylinder. This strategy is an example of one which is not identifiable but for which accurate estimates of the transformations parameters can be determined once an initial estimate of at least one of the rotation parameters is provided. These estimates are readily supplied.

3 PRACTICAL IMPLEMENTATION OF REPOSITIONING METHODS

We now consider practical ways to specify frames of reference from measurements.

3.1 REALISING REGISTRATION POINTS

In order to implement repositioning methods we need ways of defining and measuring registration points associated with an artefact.

Centres of three or more spheres

The most obvious and direct approach is to define the registration points as the centre of smooth spheres (reference balls) attached to the artefact. There are many practical advantages in using spheres as registration points:

CMM reference balls with high sphericities of various materials mounted on stalks are

Table 1: Measurement strategy for a cylindrical arrangement of 15 registration points.

Pos.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	x	x	x												
1	x	x		x	x		x	x							
2		x	x		x	x		x	x						
3	x		x	x		x	x		x						
4				x	x		x	x		x	x				
5					x	x		x	x		x	x			
6				x		x	x		x	x		x			
7							x	x		x	x		x	x	
8								x	x		x	x		x	x
9							x		x	x		x	x		x
10													x	x	x

available off the shelf. For example, one manufacturer supplies 10, 15, 20, 25 and 50 mm diameter spheres made from tungsten carbide, steel or zirconia (a ceramic) of sphericity 1.0, 0.5 or 0.25 micrometres on 50 mm stalks. The stalks have standard 6 mm threaded ends so that they can be attached easily to any artefact with tapped holes.

Spheres can be probed from any direction.

One potential disadvantage of using spheres is that software is required to determine estimates of the sphere centre from points measured on its surface. However, there are reliable algorithms and well-engineered software implementations to perform this task; see Forbes (1989), for example.

3.1.2 Three or more fixed planar surfaces

Given a perfect cuboid of known dimensions then measuring two points on each of three mutually orthogonal faces can be used to determine the position of the cuboid. Alternatively, one can measure three points on one face, two on another and one on a third. If more than six points are measured on three or more faces, then the position and orientation can be determined by a best fit to the data. If the faces are only approximately orthogonal to each other but are flat and their relative disposition is known then the same strategies can be adapted to determine position and orientation.

Three or more cylindrical dowels

Instead of using a sphere centre to determine a point in space, it is possible to use cylindrical dowels with a flat end orthogonal to the cylinder axis to do the same. Measurements of the cylindrical surface can be used to determine an estimate of the cylinder axis position and measurements on the end face can be used to determine a point on the cylinder axis. In this way, three or more cylindrical dowels can be used to determine a frame of reference. The disadvantages of this approach are that the probing and software requirements are much less straightforward than for spheres.

Clearly, combinations of spheres centres, cylindrical dowels, flat surfaces and other geometrical elements can be used to determine a frame of reference. However, for our purposes the use of reference spheres has so many advantages that no other method will be considered further.

3.2 METHODS FOR ATTACHING REGISTRATION POINTS TO AN ARTEFACT

As stated, repositioning requires that the registration points and artefact move as a rigid body. There are two basic approaches that can be adopted. Either the registration points are attached directly to the artefact or they are mounted on a jig on which the artefact is placed. We consider these cases further.

Direct attachment

Many artefacts have tapped holes or could be designed to have tapped holes. For such artefacts, reference spheres on stalks with standardised threads at one end can be mounted easily on to the artefact. The advantages of this method are:

The reference spheres and artefact assembly is close to the ideal rigid body, especially if the stalk lengths are small compared to the overall dimension of the artefact. In particular, the locations of the spheres will accurately reflect the artefact geometry during changes due to thermal effects or deflection due to bending under gravity. (In fact in some situations the changes in relative sphere locations could be used to monitor such effects.)

The same reference spheres with stalks can be used for a wide variety of artefacts.

Possible disadvantages include:

The position of the spheres is restricted to the locations of the tapped holes.

The spheres could limit access to the relevant surfaces of the artefact.

Use of jigs

For some artefacts, for example, an engine crank shaft, there is no opportunity to attach spheres directly to the artefact and supplementary jigs have to be employed. If jigs are to be employed, there are advantages in using standard jigs for classes of similar artefacts:

The jigs can be designed to have tapped holes to which the spheres can be affixed.

The spheres can be placed in a symmetrical pattern on the jig so that a) the CNC programming for the CMM is simplified through step-and-repeat paradigms, and b) the spheres are measured in roughly the same locations within the working volume of the CMM. This means that the CMM's systematic errors (which can vary with location) have less effect in the determination of the frame of reference.

The sphere locations can be calibrated before the artefact is mounted and this additional calibration information incorporated into the data analysis. (This assumes that the jig assembly does not distort after the artefact is affixed to the jig.)

CMM control software for probing the spheres can be more readily reused.

There are also disadvantages in the use of jigs:

Jigs can restrict access to the artefact and also restrict the positions in which the artefact can be placed.

Care has to taken to ensure that the artefact mounted on the jig does not suffer any additional stress by being overly constrained. To minimise difficulties with this, it is recommended that the artefact and jig should be made of materials with similar thermal expansion properties.

Example: rotary manipulator with spheres attached. One system implemented at NPL uses a rotary table of approximate diameter 400 mm to which six reference spheres have been attached in a hexagonal pattern around the circumference. The positions of the six spheres have been calibrated so that, for any subsequent experiments, measurements of the position of any three spheres are enough to determine the frame of reference. An artefact can be mounted in the centre of the table and rotated by any angle. Such a system can be used to measure API standards, cam shafts or any other artefact for which a rotary table is usually employed. There is no need for the rotary manipulator to have a high specification since the position of the artefact is determined from the measurements of the spheres. In this way a standard CMM can have much of the added functionality of a rotary table without great additional expense.

Example: linear staging jig. A second example implemented at NPL is a 1 m long jig to which are attached ten reference spheres in the 5×2 grid described above. The spacing between the spheres is 200 mm on each axis. This jig has been used successfully to calibrate a 1 m long step gauge using repositioning (see section 4.1 below).

Example: clamping jigs. For artefacts such as a crankshaft which have a well defined shaft, an obvious method for attaching reference spheres is to use a two-part clamp which can be fixed on to the shaft. Reference spheres can be screwed into tapped holes in the clamp. This method can be used along with the cylindrical artefact measurement strategy described above; see Butler, Corta *et al.* (1997).

3.3 METHODS FOR REPOSITIONING ARTEFACTS

In this section we consider methods for translating and rotating artefacts within the working volume of a CMM. Three motions are considered: rotation about a vertical axis, translation about a horizontal axis and rotation along a horizontal axis.

Rotation about a vertical axis

This is most readily achieved using a rotary manipulator discussed already. If required the artefact can be clamped or fixed directly to the rotary table using the tapped holes of the table. For some artefacts adhesive wax will be sufficient to maintain rigidity of the artefact-table assembly. The limitation of this method is likely to be the vertical axis travel.

Translation along a horizontal axis

This type of motion is required for calibrating artefacts longer than the longest axis of a CMM. One method implemented at NPL for lighter artefacts is to mount the artefact on a long jig that can be fixed to the CMM table in various positions overhanging at either end of the CMM table. The jig and artefact assembly is simply manipulated from one position to another. This was the approach adopted in the calibration of the 1 m step gauge (section 4.1).

An extension of this method suitable for heavier artefacts is to attach parallel tracks to the CMM table on which two or more small bogeys can run and be locked into position. The bogeys can either be part of one assembly or move independently. The artefact is mounted on the bogeys and can be translated along the track.

Rotation about a horizontal axis

This type of motion is required, for example, when calibrating cylindrical-type artefacts that are too long to be mounted vertically.

One method implemented at NPL for lighter artefacts is simply to remount the artefact in the various orientations; this was the approach adopted for the calibration of shell-like artefacts. For heavier artefacts, the artefact can be mounted on two supports each of which have bearings on which the artefact can be rotated and then locked into position. The Centro Español de Metrologia (CEM) and its collaborator Tekniker (Butler, Corta *et al.*, 1997) have implemented a method that combines both translation along and rotation about a horizontal axis. The approach involves two bogeys running on a track, each of which have bearings.

3.4 POSITION CALIBRATION SOFTWARE

Repositioning also depends on being able to determine accurately the transformations which link the various frames of reference. The mathematical model described above leads to an unconstrained nonlinear least squares problem and can be solved by a Gauss-Newton-type algorithm (Gill, Murray and Wright, 1981). A comprehensive software package[1] for solving this problem has been developed by NPL and is fully described in Butler, Forbes and Kenward (1997).

The main features of the software are:

Portability. The software is written in ANSI Standard Fortran 77 and has been tested for portability using the Toolpack portability verifier (NAG, 1992) on a DEC Alpha running VMS.

Scope. The subroutines cater for an arbitrary number of points measured in an arbitrary number of positions in any order. In practice, the maximum size of the problem will be determined by the user's driver program which will define the maximum array sizes.

Computational requirements. The main optimisation algorithm implemented has $O(m_X n_T^2)$ performance, where m_X is the total number of measurements and n_T the number of transformations $\{t_k\}$. Efficiency of the software is enhanced by the use of structure exploiting techniques to solve the matrix equations arising in the optimisation process (Cox, 1990).

Memory requirements. The software requires $O((n_Y+n_T)n_T)$ memory, including work space, where n_Y is the total number of registration points.

Software validation. The main optimisation modules have been tested on reference data and results generated using null space methods (Butler, Cox *et al*., 1997, Forbes, 1994). The tests show that the software is able to determine estimates of the transformation parameters without significant loss of precision.

Initial estimation of the parameters. The software determines initial estimates of all the optimisation parameters. All the user has to supply is the measurement data along with indices indicating the registration point and frame of reference corresponding to a measured point.

Consistency checking. The software checks for consistency of the data by monitoring the apparent distance between registration points as measured in different frames of reference.

Statistics. Standard uncertainties of the fitted parameters and related quantities are calculated in accordance with the ISO Guide (ISO, 1993).

The main modules have also been implemented in Matlab (Matlab, 1992).

[1] For details about the availability of this package contact A B Forbes, NPL.

3.5 LIMITATIONS OF THE REPOSITIONING METHOD

There are a number of practical factors which limit the application of repositioning methods.

Artefact deflection. As an artefact changes position and orientation, it is possible for the artefact to change shape due to gravity.

Weight. Compact CMMs are not designed for large heavy artefacts. Typically a small fixed-table CMM might handle artefacts up to 500 kg, while a similarly sized moving-table CMM may be limited to artefacts up to 100 kg. The option of supporting the artefact by some external method is not practical as it would introduce relatively large influences into the measuring environment.

Overhang. There is also a limit on the weight which can overhang from the CMM table.

Access. Depending on the architecture of the CMM, one, two or three sides of the CMM will have limited access and this will restrict the type of repositioning that can be performed.

Some of the limitations described above could be removed through different CMM design. For example, we could design a moving bridge CMM with a working volume of 1 m cubed, but with a 5 m × 1 m fixed table. Repositioning methods could be used to provide an effective working volume of 5 m × 1 m × 1 m (or more).

4 MEASUREMENT EXPERIMENTS

In order to illustrate the methodology, we describe three measurement problems undertaken at NPL using repositioning. All of the measurements were taken on NPL's Zeiss UPMC 550 which has a longest measuring axis of 550 mm. The 3D length measuring capability of the CMM as determined by a CMM verification according to ISO 10360-2 (ISO 10360-2) is ±(1.3 + L/400).

4.1 THE STEP GAUGE EXPERIMENT

The aim of this experiment was to use repositioning to measure a 1 m step gauge that had been calibrated recently at NPL. The step gauge was mounted along a steel frame to which were attached ten zirconia spheres of diameter 20 mm and sphericity 1 micrometre. The spheres were positioned approximately on a 5 × 2 grid, 5 spheres on one side of the step gauge, 5 spheres on the other side, with the spheres numbered as in section 2.2.3.

The step gauge was measured in four positions, corresponding to the initial position and then three translations approximately in line with the axis of the step gauge. Measurements associated with the following information were recorded.

Position	Sphere centres	Faces
0	1, 2, 3, 4 and 6	000, 100, 200, 300, 400 and 500
1	1, 2, 3, 4, 5 and 6	200, 300, 400 and 500
2	3, 4, 5, 6, 7 and 8	400, 500, 600 and 700
3	5, 6, 7, 8, 9 and 10	600, 700, 800 and 900

The complete experiment was repeated with the artefact and its mounting rotated through approximately 180°. We will distinguish the two experiments by a "minus-plus" or "plus-minus" label.

Table 2 shows how the step positions calculated by the repositioning method, minus-plus orientation, compares with the reference results determined from the step gauge calibration. The equivalent results for the plus-minus orientation are given in Table 3.

Looking at Table 2 first, we see that the largest discrepancy is 2.0 micrometres at the 500 mm face. The two sets of measurements agree at the 900 mm face to 0.6 micrometres, better than 1 part in 10^6. To analyse the errors introduced by the repositioning, we note firstly that the 2 micrometre discrepancy at the 500 mm face is already apparent from the measurement of the step gauge in the initial (0th) position. This is well within the accuracy expected of the CMM: the measurements span 500 mm and the length measuring capability of the CMM for this experiment is of the order of

$$\pm(1.3 + 500/400) = \pm 2.5$$

micrometres. Thus, this difference is within the limits associated with the CMM systematic error. The second point to note is that the positions of the step faces determined from the repositioning measurements are reasonably consistent. The largest spread of 0.8 micrometres occurs at the 300 mm face.

The measurement results in Table 3 for the plus-minus orientation are broadly similar to those for the minus-plus orientation as seen in Table 4 below. Note the good agreement (0.6 micrometres or better) between the two repositioning experiment results. This analysis does not take into account systematic CMM error, measurement error in the calibration of the step gauge and systematic differences introduced by any small difference in the alignment of the measurements along the step gauge.

Table 2. Measurements of the step positions (in mm) in the step gauge experiment for the minus-plus orientation. The second column gives the reference locations determined from the step gauge calibration while the remaining five columns give the locations as determined by the repositioning method for the step gauge in the four positions, along with their mean.

Face	Refer.	Pos. 0	Pos. 1	Pos. 2	Pos. 3	Mean
000	000.0000	000.0000				0
100	100.0000	100.0001				100.0001
200	199.9994	199.9989	199.9983			199.9986
300	299.9993	299.9981	299.9973			299.9977
400	399.9996	399.9984	399.9979	399.9977		399.9980
500	499.9994	499.9974	499.9973	499.9974		499.9974
600	599.9994			599.9976	599.9977	599.9977
700	699.9988			699.9976	699.9978	699.9977
800	799.9985				799.9989	799.9989
900	899.9991				899.9985	899.9985

Table 3. As Table 2 but for the plus-minus orientation.

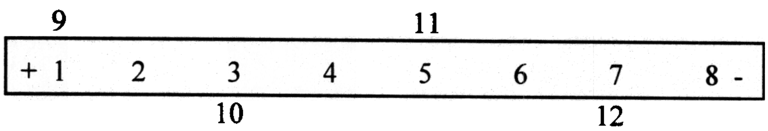
Face	Refer.	Pos. 0	Pos. 1	Pos. 2	Pos. 3	Mean
000	000.0000	000.0000				0
100	100.0000	100.0003				100.0003
200	199.9994	199.9988	199.9993			199.9991
300	299.9993	299.9982	299.9976			299.9979
400	399.9996	399.9987	399.9983	399.9987		399.9986
500	499.9994	499.9986	499.9973	499.9978		499.9979
600	599.9994			599.9981	599.9979	599.9980
700	699.9988			699.9981	699.9981	699.9981
800	799.9985				799.9990	799.9990
900	899.9991				899.9984	899.9984

Table 4. Comparison of step positions (in mm) for the step gauge experiment. The second column gives the reference locations determined by the gauge calibration, the third gives the average location determined by the repositioning experiment for the minus-plus orientation, while the fourth column has the corresponding results for the plus-minus orientation repositioning experiment.

Face	Reference	Minus-plus	Plus-minus
000	000.0000	000.0000	000.0000
100	100.0000	100.0001	100.0003
200	199.9994	199.9986	199.9991
300	299.9993	299.9977	299.9979
400	399.9996	399.9980	399.9986
500	499.9994	499.9974	499.9979
600	599.9994	599.9977	599.9980
700	699.9988	699.9977	699.9981
800	799.9985	799.9989	799.9990
900	899.9991	899.9985	899.9984

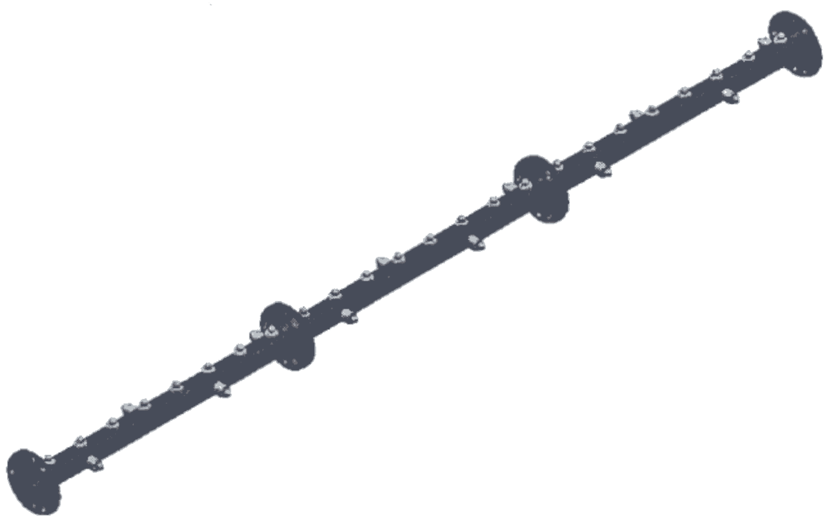
4.2 LARGE ARTEFACT MODULE CALIBRATION

NPL and collaborators CEM, Spain and the National Metrology Laboratory (NML), Ireland, have designed and constructed a large reference artefact for CMM verification (Forbes and Peggs, 1997, Butler, Corta, *et al.*, 1997). The artefact is assembled from 4 × 1.0 m and 2 × 0.5 m tubular sections made from carbon fibre. The modules can be locked together semi-kinematically through carbon fibre flanges to make a composite artefact of up 5.0 m long; see Figure 1. NPL was required to measure the positions of the centres of 12 ceramic references spheres attached to each of the 1.0 m modules. The positions of the reference spheres on each of the 1 m modules is indicated below.



Each module was mounted on an aluminium frame, supported by two vee-blocks positioned under spheres 2 and 7. Each module was measured in three positions, corresponding to the initial

Figure 1. Three 1 m modules of a large reference artefact assemble in line. Each module has eight reference spheres mounted on top and two on each side.



position and then two translations approximately in line with the axis of the tubular section:

Position	Sphere centres
0	1, 2, 3, 4, 9 and 10
1	3, 4, 5, 6, 10 and 11
2	5, 6, 7, 8, 11 and 12

Table 5 shows the distances (in mm) between selected pairs of reference spheres on one of the modules, along with estimates of the corresponding standard uncertainties (with coverage factor $k = 1$). The estimate of the standard deviation of the measurement errors was $\hat{\sigma} = 0.18$ micrometres.

The fit of the model to the data indicated by the standard deviation of the measurements errors is exceptionally good; for other modules $\hat{\sigma} \approx 0.35$ micrometres. The standard uncertainties associated with the distances are also extremely small, reflecting the consistency of the model with the data.

Table 5. Distances and associated standard uncertainties calculated from measurements of one 1 m module of the large reference artefact. The units are millimetres.

Indices,		distances and standard	uncertainties:
1	2	124.93580	0.00011
1	3	250.08118	0.00010
1	4	375.15502	0.00010
1	5	499.87284	0.00013
1	6	624.95344	0.00013
1	7	749.88709	0.00015
1	8	875.02706	0.00015
2	8	750.09292	0.00015
3	8	624.94603	0.00013
4	8	499.87204	0.00013
5	8	375.15430	0.00010
6	8	250.07380	0.00010
7	8	125.14174	0.00011

4.3 MEASUREMENT OF A 5 M REFERENCE ARTEFACT

As an illustration of the scope of the repositioning methods we also present results for the measurement of all modules of the 5 m reference artefact. The measurements of two of the modules can be related to each other by measuring the reference spheres 7, 8, 11, and 12 of one module and spheres 1, 2, 9 and 10 of the second module while the two modules are locked together. The complete measurement of the 5 m artefact requires a minimum of 19 positions to measure all 60 reference spheres on the NPL Zeiss UPMC 550. Table 6 shows the distances (in mm) between selected pairs of reference spheres along with estimates of the corresponding uncertainties (with coverage factor k = 1). The estimate of the standard deviation of the measurement errors was σ = 0.41 micrometres. The results show that the maximum standard uncertainty is 0.75 micrometres associated with the longest length of 4.825 m, i.e., better than 2 parts in 10^7. This standard uncertainty takes no account of systematic errors associated with the CMM or artefact but is an indication of the consistency of the model with the data and reflects the uncertainty associated with the repositioning method. Since these uncertainties are very much smaller than those accounting for other measurement error, we can conclude that the repositioning method can be used to measure artefacts considerably longer than the working dimensions of the CMM without introducing significant error. These measurements have also been repeated at CEM and NML.

4 CONCLUDING REMARKS

This report has described a repositioning methodology and considered its practical implementation. Measurement experiments have confirmed the effectiveness of the approach which is now common practice at NPL. The scope of the method is indicated by the calibration

Table 6. Distances and associated standard uncertainties calculated from measurements of all six modules of the large reference artefact along with measurements of the five joins. The units are millimetres.

Indices,		distances and standard	uncertainties
1	7	499.93062	0.00029
1	11	1000.08830	0.00037
1	19	1500.00147	0.00044
1	23	1999.90176	0.00050
1	31	2499.91972	0.00055
1	35	2999.92930	0.00060
1	43	3500.04718	0.00064
1	47	3999.91974	0.00068
1	55	4500.21015	0.00072
1	58	4875.16231	0.00075
4	58	4500.18335	0.00072
10	58	4000.19567	0.00068
14	58	3500.08227	0.00064
22	58	3000.20848	0.00060
26	58	2500.07877	0.00055
34	58	2000.19163	0.00050
38	58	1500.27701	0.00044
46	58	999.96035	0.00037
50	58	500.08840	0.00029

of a 5 m artefact on a CMM with a longest measuring axis of 550 mm. However, increasing the effective measuring capacity of a CMM by even 50% is of significant value. Repositioning can also offer major advantages in the calibration of complex artefacts where complicated probing arrangements would otherwise be necessary.

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